Sample Objective Type Questions–Mathematics

1. $(2^{\log_{1/3}\frac{1}{8}})^{\log_4 3}$ equals

   (A) 1
   (B) 2
   (C) $2\sqrt{2}$
   (D) 4
   (E) 8

2. The sum of all odd integers greater than 100 and less than 1000 is

   (A) 247500
   (B) 495000
   (C) 990000
   (D) 1100000
   (E) None of the above.

3. The value of a such that the planes $ax+y+z=0$, $x+3z=0$, and $5y+6z=0$ have a line in common is

   (A) -1/15
   (B) -11/15
   (C) 11/15
   (D) 1/15
   (E) 0

4. The set of real x for which $(x+1)^3(x-2)^2(x-3)<0$ is

   (A) $(-1,3)$
   (B) $(-\infty,-1)\cup(3,\infty)$
   (C) $(-1,\infty)$
   (D) $(-\infty,3)$
   (E) $(-1,2)\cup(2,3)$

5. $\lim\limits_{x\to\pi}\dfrac{\tan x}{\sin(\frac{\pi-x}{2})}$ equals

   (A) 0
   (B) $1/2$
   (C) 2
   (D) -1
   (E) -2

6. If the surface area of a cube is decreased by 19%, the volume of the cube decreases by

   (A) 9%
   (B) 19%
   (C) 27.1%
   (D) 37.1%
   (E) 85.2%

7. Each vertex of a cube is assigned an integer so that no two vertices connected by an edge of the cube are assigned the same integer. The least number of integers required for such an assignment is

   (A) 2
   (B) 3
   (C) 4
   (D) 6
   (E) 8

8. Let f and g be functions such that $f(x)=f(-x)$, $g(x)=-g(-x)$, $\int_0^2 f(x)\,dx=7$, and $\int_0^2 g(x)dx=3$. Calculate $\int_{-2}^{2}(f(x)+g(x))dx$.

(A) 6
(B) 10
(C) 14
(D) 20
(E) Cannot be determined from the given data.

9. Which is the smallest number with exactly 12 divisors? (If n is a positive integer, 1 and n are counted as divisors of n. So, for example, 4 has three divisors: 1, 2, and 4.)

(A) 72
(B) 2^{11}
(C) 12
(D) 48
(E) None of the above.

10. If α, β are the roots of $(x-2)(x-\sqrt{3}) = \sqrt{7}$, the roots of $(x-\alpha)(x-\beta)+\sqrt{7} = 0$ are

(A) $-2-\sqrt{3}$, $\sqrt{7}$
(B) $2-\sqrt{7}$, $\sqrt{3}-\sqrt{7}$
(C) $2+\sqrt{3}$, $\sqrt{7}$
(D) $2+\sqrt{7}$, $\sqrt{3}+\sqrt{7}$
(E) 2, $\sqrt{3}$

Answers: 1. C, 2. A, 3. A, 4. E, 5. E, 6. C, 7. A, 8. C, 9. E (The correct number is 60), 10. E

Long Answer Type Questions

1. If $(1+x)^n = c_0 + c_1x + c_2x^2 + \ldots + c_nx^n$, show that (8 marks)

$$c_0^2+2c_1^2+\cdots+(n+1)c_n^2 = \frac{(2n-1)!\,(n+2)}{n!(n-1)!}$$

2. Find the centre and radius of the circle that is the intersection of the spheres

$$x^2+y^2+z^2-8=0$$

and

$$x^2+y^2+z^2-4x-4y-2z+4=0$$

(10 marks)

3. What is the probability that a point chosen at random within a square is closer to the centre than to the boundary? (12 marks)

4. Characterize all arithmetic progressions in positive integers such that, for all $n \geq 1$, the sum up to n terms is a perfect square. (12 marks)