Example 11-1: Forward Fourier Transform

As an example of determining the Fourier transform, consider the one-sided exponential signal

$$x(t) = e^{-7t}u(t) \tag{11.4}$$

Substituting this function into (11.1) gives

$$\begin{aligned} X(j\omega) &= \int_0^\infty e^{-7t} e^{-j\omega t}\,dt \\ &= \int_0^\infty e^{-(7+j\omega)t}\,dt \\ &= \left.\frac{e^{-7t}e^{-j\omega t}}{-(7+j\omega)}\right|_0^\infty \\ &= \frac{1}{7+j\omega} \end{aligned} \tag{11.5}$$

Note that the value of the integral at the upper limit is zero because the magnitude of $e^{-7t}e^{-j\omega t}$ is equal to e^{-7t} which goes to zero as $t \to \infty$.[1] Thus, we have derived the following Fourier transform pair (in the notation of (11.3)):

Time-Domain		***Frequency-Domain***
$e^{-7t}u(t)$	$\stackrel{\mathcal{F}}{\longleftrightarrow}$	$\dfrac{1}{7+j\omega}$

(11.6)

Notice that in evaluating the integral in (11.5) no restrictions were placed on ω, so we conclude that all frequencies, $-\infty < \omega < \infty$, are required to represent $e^{-7t}u(t)$ by the Fourier transform. ■

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[1] Recall $|e^{-7t}e^{-j\omega t}| = |e^{-7t}||e^{-j\omega t}|$, and $|e^{-j\omega t}| = 1$.