Example 11-12: Derivative Property

Suppose that we want to find the Fourier transform of the derivative of the signal in Example 11-11; i.e.,

$$y(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left(e^{-2t}u(t-3)\right)$$

One approach would be to differentiate the signal and then determine the Fourier transform of the result. Using the product rule, the derivative is

$$\begin{aligned} y(t) &= e^{-2t}\delta(t-3) - 2e^{-2t}u(t-3) \\ &= e^{-6}\delta(t-3) - 2e^{-6}e^{-2(t-3)}u(t-3) \end{aligned}$$

Using results from Example 11-11 on p. 11-11 and the Fourier transform of a delayed impulse from (11.76) on p. 11.76, we obtain

$$Y(j\omega) = e^{-6}e^{-j\omega 3} - \frac{2e^{-6}e^{-j\omega 3}}{2+j\omega}$$

Placing the whole expression over a common denominator gives a form that shows we could have just multiplied $X(j\omega)$ in (11.78) by $(j\omega)$ since

$$Y(j\omega) = \frac{e^{-6}(j\omega)e^{-j\omega 3}}{2+j\omega} = j\omega X(j\omega)$$

In other words, we would have obtained the same answer by applying the differentiation property (11.82) directly to the result of Example 11-11. ■

McClellan, Schafer and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.