

Example 11-13: Transform a Differential Equation

Likewise, if we were given a differential equation, we could use the differentiation property to take the Fourier transform of both sides of the differential equation. Starting with

$$\frac{d^2 y(t)}{dt^2} + 2\frac{dy(t)}{dt} = \frac{dx(t)}{dt} + 3x(t)$$

we would obtain

$$(j\omega)^2 Y(j\omega) + 2(j\omega)Y(j\omega) = (j\omega)X(j\omega) + 3X(j\omega)$$

If we factor out $Y(j\omega)$ on the left-hand side and $X(j\omega)$ on the right-hand side, we obtain

$$[(j\omega)^2 + 2(j\omega)]Y(j\omega) = [(j\omega) + 3]X(j\omega)$$

which can be solved for $H(j\omega)$ as

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{(j\omega) + 3}{(j\omega)^2 + 2(j\omega)}$$

McClellan, Schafer and Yoder, Signal Processing First, ISBN 0-13-065562-7.

Prentice Hall, Upper Saddle River, NJ 07458. © 2003 Pearson Education, Inc.