Example 11-2: Unique Inverse

Consider the problem of evaluating the following integral:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{1}{7+j\omega}\right)e^{-j3\omega}\,d\omega = ? \tag{11.7}$$

This integral is difficult, if not impossible, to evaluate by ordinary methods of integral calculus. However, the integral is a special case of an inverse transform integral, so the uniqueness of the Fourier transform representation guarantees that we can be confident in writing

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{1}{7+j\omega}\right)e^{j\omega t}\,d\omega = e^{-7t}u(t) \tag{11.8}$$

All we have to do is remember the Fourier transform pair in (11.6). Uniqueness guarantees that there is only one time function that goes with a given Fourier transform. Finally, we can "do the integral" in (11.7) by taking the special case of $t = -3$ in (11.8) which means that we evaluate $e^{-7t}u(t)$ at $t = -3$ to get the answer of zero! It would be very difficult to obtain this answer by the ordinary methods of integral calculus. ■

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