Example 11-3: Square Wave Transform

Figure 11-9 shows a periodic square wave where $T_0 = 2T$. If we substitute this function into (11.34) we obtain

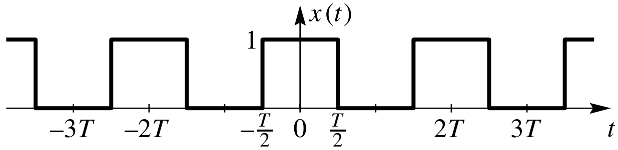

Figure 11-9: Signal $x(t)$ is a 50% duty cycle square wave whose period is $T_0 = 2T$. Its transform is shown in Fig. 11-8.

$$
\begin{aligned}
a_k &= \frac{1}{T_0}\int_{-T/2}^{T/2} e^{-jk\omega_0 t}\,dt \\
&= \frac{1}{T_0}\frac{e^{-jk\omega_0 t}}{(-jk\omega_0)}\Bigg|_{-T/2}^{T/2} = \frac{\sin(k\omega_0 T/2)}{k\omega_0 T_0/2}
\end{aligned} \tag{11.37}
$$

where $\omega_0 = 2\pi/T_0$. We also obtain the DC coefficient by evaluating the integral

$$
a_0 = \frac{1}{T_0}\int_{-T/2}^{T/2} dt = \frac{T}{T_0} = \frac{1}{2} \tag{11.38}
$$

After substituting $(\omega_0 T) = (2\pi/T_0)(T_0/2) = \pi$ into (11.37), we obtain

$$
a_k = \begin{cases} \dfrac{\sin(\pi k/2)}{\pi k} & k \neq 0 \\ \frac{1}{2} & k = 0 \end{cases} \tag{11.39}
$$

If we substitute (11.39) into (11.35) we obtain the following equation for the Fourier transform of a periodic square wave:

$$
X(j\omega) = \pi\delta(\omega) + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \left(\frac{2\sin(\pi k/2)}{k}\right)\delta(\omega - k\omega_0) \tag{11.40}
$$

Figure 11-8 shows the Fourier transform of the square wave for the case $T_0 = 2T$. The Fourier coefficients are zero for even multiples of ω_0, so there are no impulses at those frequencies. Any periodic signal with fundamental frequency ω_0 will have a transform with a similar appearance—impulses at integer multiples of ω_0, but with different sizes dictated by the a_k coefficients. ■

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