Example 11-4: Transform of Impulse Train

As another example of finding the Fourier transform of a periodic signal, let us consider the periodic impulse train

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \tag{11.41}$$

where the period is denoted by T_s. This signal, which will be useful in Chapter 12 in deriving the sampling theorem, is plotted in Fig. 11-10(a). Because $x(t)$ is periodic with period T_s, we can also express (11.41) as a Fourier series

$$p(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_s t} \tag{11.42}$$

where $\omega_s = 2\pi/T_s$. To determine the Fourier coefficients $\{a_k\}$, we must evaluate the Fourier series integral over one

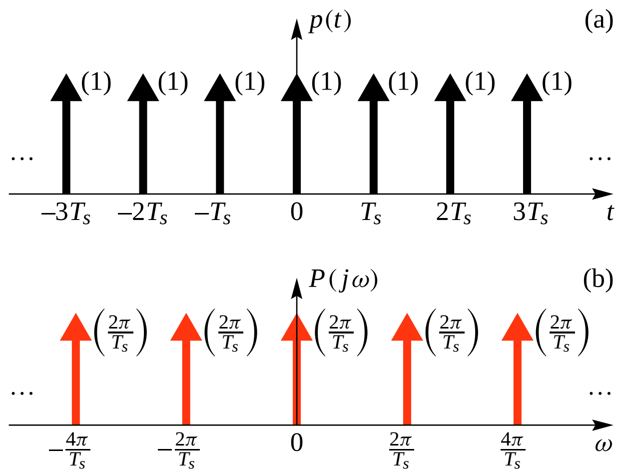

Figure 11-10: Periodic impulse train: (a) Time-domain signal $p(t)$; (b) Fourier transform $P(j\omega)$. Regular spacing in the frequency-domain is $\omega_s = 2\pi/T_s$

convenient period; i.e.,

$$\begin{aligned} a_k &= \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} \delta(t) e^{-jk\omega_s t} dt \\ &= \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} \delta(t) dt = \frac{1}{T_s} \end{aligned} \tag{11.43}$$

The Fourier coefficients for the periodic impulse train are all the same size. Now in general, the Fourier transform of a periodic signal represented by a Fourier series as in (11.42) is of the form

$$P(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_s)$$

Substituting (11.43) into the general expression for $P(j\omega)$, we obtain

$$P(j\omega) = \sum_{k=-\infty}^{\infty} \left(\frac{2\pi}{T_s}\right) \delta(\omega - k\omega_s) \tag{11.44}$$

Therefore, the Fourier transform of a periodic impulse train is also a periodic impulse train. ■

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.