Example 7-11: $H(e^{j\hat{\omega}})$ from $H(z)$

For a 10-point running-sum filter ($L = 10$), the system function is

$$H(z) = \sum_{k=0}^{9} z^{-k} = \frac{1 - z^{-10}}{1 - z^{-1}} = \frac{z^{10} - 1}{z^9(z-1)} \tag{7.38}$$

A pole-zero diagram for this case is shown in Fig. 7-11, and the corresponding frequency response for the running-sum filter is shown in Fig. 7-12. The factors of the numerator are the tenth roots of unity, and the zero at $z = 1$ is canceled by the corresponding term in the denominator. This explains why we have only nine zeros around the unit circle with the gap at $z = 1$. The nine zeros around the unit circle in Fig 7-11 show up as zeros along the $\hat{\omega}$ axis in Fig. 7-12 at $\hat{\omega} = 2\pi k/10$, and it is the gap at $z = 1$ that allows the frequency response to be larger at $\hat{\omega} = 0$. The other zeros around the unit circle keep $H(e^{j\hat{\omega}})$ small, thereby creating the "lowpass" filter frequency response shown in Fig. 7-12.

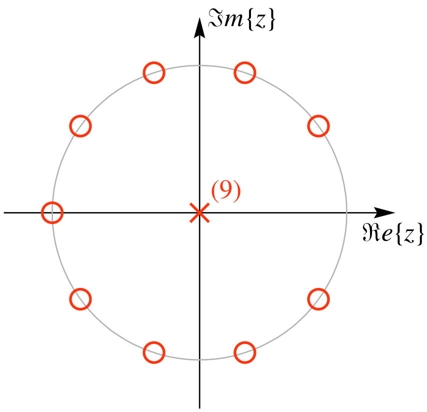

Figure 7-11: Zero and pole distribution for the 10-point running-sum filter. There are nine zeros spread out uniformly along the unit circle, and nine poles at the origin.

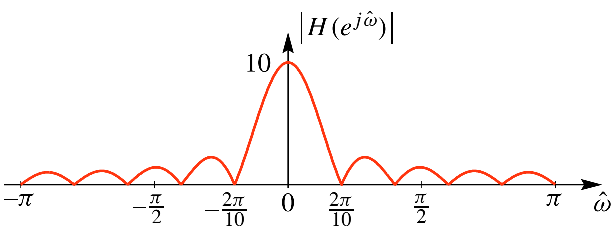

Figure 7-12: Frequency response (magnitude only) for the 10-point running-sum filter. These are the values along the unit circle in the z-plane. There are nine zeros spread out uniformly along the frequency axis.

McClellan, Schafer and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.