Example 7-7: Split $H(z)$ into Cascade

Consider the following example

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

One of the roots of $H(z)$ is $z = 1$, so $H_1(z) = (1 - z^{-1})$ is a factor of $H(z)$. The other factor can be obtained by division

$$H_2(z) = \frac{H(z)}{H_1(z)} = \frac{H(z)}{1 - z^{-1}} = 1 - z^{-1} + z^{-2}$$

The factorization of $H(z)$ as

$$H(z) = (1 - z^{-1})(1 - z^{-1} + z^{-2})$$

gives the cascade shown in the block diagram of Fig. 7-3. The resulting difference equations for the cascade are

$$w[n] = x[n] - x[n-1]$$
$$y[n] = w[n] - w[n-1] + w[n-2]$$

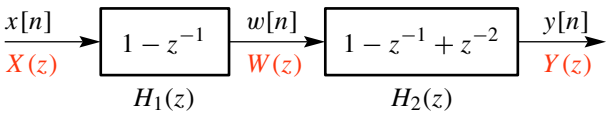

Figure 7-3: Factoring $H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$ into the product of a first-order system and a second-order system.

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