Example 7-9: Zeros and Poles of $H(z)$

Consider the system function

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3},$$

which can be expressed in the following different forms:

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3} \quad (7.27)$$

$$= (1 - z^{-1})(1 - e^{j\pi/3}z^{-1})(1 - e^{-j\pi/3}z^{-1}) \quad (7.28)$$

or, if we multiply $H(z)$ by z^3/z^3, we obtain the following two equivalent forms:

$$H(z) = \frac{z^3 - 2z^2 + 2z - 1}{z^3} \quad (7.29)$$

$$= \frac{(z-1)(z - e^{j\pi/3})(z - e^{-j\pi/3})}{z^3} \quad (7.30)$$

Equations (7.27)–(7.30) give four different equivalent forms of $H(z)$. The factored form in (7.30) shows clearly that the zeros of $H(z)$ are at locations $z_1 = 1$, $z_2 = e^{j\pi/3}$, and $z_3 = e^{-j\pi/3} = z_2^*$ in the z-plane. Equation (7.30) also shows that $H(z) \to \infty$ for $z \to 0$. Values of z for which $H(z)$ is undefined (infinite) are called ***poles*** of $H(z)$. In this case, we say that the term z^3 represents three poles at $z = 0$ or that $H(z)$ has a third-order pole at $z = 0$. We have stated that the poles and zeros determine the system function to within a constant. As an illustration, note that the polynomial $\frac{1}{2}H(z) = 0.5 - z^{-1} + z^{-2} - 0.5z^{-3}$ has exactly the same poles and zeros as $H(z)$ in (7.27). ■

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