Example 9-12: Impulse Convolution Causes Shifting

When one of the signals in a convolution is a regular continuous-time function and the other contains only impulses, a graphical approach often leads to a simple solution. Consider the pulse input $x(t)$ and the impulse response $h(t) = \delta(t-1) + 0.5\delta(t-2)$ shown in Fig. 9-11. The equation for the output is simply

$$\begin{aligned} y(t) &= x(t) * [\delta(t-1) + 0.5\delta(t-2)] \\ &= x(t-1) + 0.5x(t-2) \end{aligned}$$

As Fig. 9-11 shows, such convolutions are easy to do graphically. We simply shift a scaled copy of the continuous signal to the location of each impulse and sum all the shifted and scaled copies. As shown by the dotted lines in Fig. 9-11(c), the two copies overlap in the region $2 \leq t < 3$, so the output is $1 + 0.5 = 1.5$ in that interval. ■

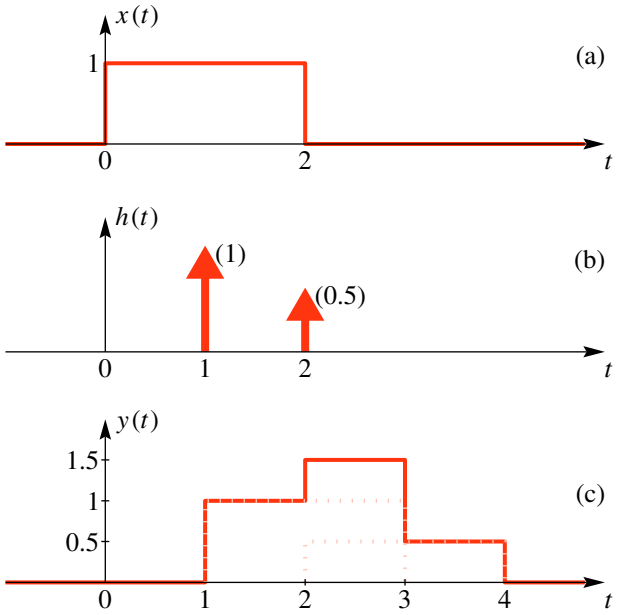

Figure 9-11: Convolution of a pulse with impulses: (a) Pulse input $x(t)$. (b) Impulse response $h(t)$ consisting only of impulses. (c) Resulting output after convolution, $y(t) = x(t) * h(t)$.

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