Example 9-15: Derivative-Integral Cascade

Consider the input and impulse response in Section 9-7.3 and plotted in Fig. 9-18 on 9-18. If we differentiate the input and integrate the impulse response before convolution, we will achieve the same result as convolving the two functions directly. In this case, $x(t) = u(t-1) - u(t-2)$ so $x^{(1)}(t) = \delta(t-1) - \delta(t-2)$. To integrate the impulse response, we convolve with the unit-step function and find that $h^{(-1)} = 0$ for $t < 0$ and

$$\begin{aligned} h^{(-1)}(t) &= \int_0^t e^{-\tau} d\tau \\ &= (1 - e^{-t}) \quad \text{for } t > 0 \end{aligned}$$

More compactly, $h^{(-1)}(t) = (1 - e^{-t})u(t)$. Now we compute $y(t)$ as follows:

$$\begin{aligned} y(t) &= x^{(1)}(t) * h^{(-1)}(t) \\ &= [\delta(t-1) - \delta(t-2)] * [(1 - e^{-t})u(t)] \\ &= (1 - e^{-(t-1)})u(t-1) - (1 - e^{-(t-2)})u(t-2) \end{aligned}$$

If we look closely at the last expression above, we see that

$$y(t) = \begin{cases} 0 & t < 1 \\ (1 - e^{-(t-1)}) & 1 \leq t < 2 \\ e^{-(t-2)} - e^{-(t-1)} & 2 \leq t \end{cases}$$

which is identical to (9.59) in Section 9-7.3. ■

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