



Example 9-3: Derivative of a Discontinuous Function

Consider the signal $x(t) = e^{-2(t-1)}u(t-1)$ which is discontinuous at $t = 1$. Its derivative is

$$\begin{aligned}\frac{dx(t)}{dt} &= e^{-2(t-1)}\frac{du(t-1)}{dt} + u(t-1)\frac{de^{-2(t-1)}}{dt} \\ &= e^{-2(t-1)}\delta(t-1) - 2e^{-2(t-1)}u(t-1) \\ &= e^0\delta(t-1) - 2e^{-2(t-1)}u(t-1)\end{aligned}$$

The graphical representation of this example is shown in Fig. 9-9. Such figures are useful because they show that each discontinuity of the function generates an impulse in the derivative whose size is equal to the size of the discontinuity of the function. ■

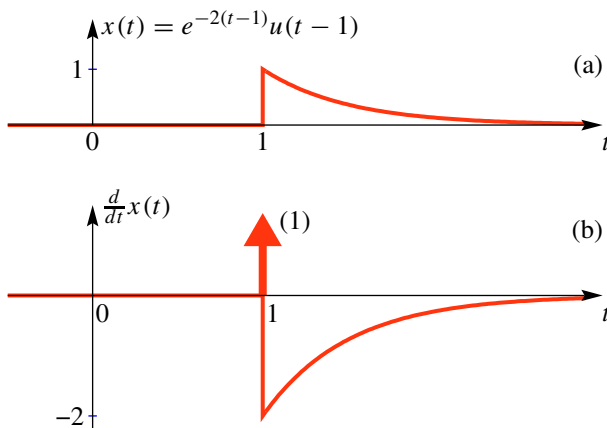


Figure 9-9: (a) Delayed one-sided exponential and (b) its derivative.