Example 9-5: Integrator is Time-Invariant

The integrator system is also a time-invariant system. To prove this, we replace $x(\tau)$ in (9.30) by $x(\tau - t_0)$ obtaining the output $w(t)$

$$w(t) = \int_{-\infty}^{t} x(\tau - t_0)d\tau \tag{9.35}$$

Now, to prove time-invariance, we must manipulate the integral in (9.35) into a form that is recognizable in terms of the original output $y(t)$. This is done by changing the "dummy variable" of integration to $\sigma = \tau - t_0$. In this substitution, $d\tau$ is replaced by $d\sigma$, the lower limit $\tau = -\infty$ becomes $\sigma = -\infty$, and the upper limit $\tau = t$ becomes $\sigma = t - t_0$. Therefore (9.35) becomes

$$w(t) = \int_{-\infty}^{t-t_0} x(\sigma)d\sigma$$

and it is now clear that $w(t) = y(t - t_0)$, so the integrator system is seen to be time-invariant. ■