

Example 9-9: Convolve Unit Steps

When the impulse response is a unit step, h(t) = u(t), and the input is also a unit step, x(t) = u(t), the convolution integral becomes

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

=
$$\int_{-\infty}^{\infty} u(\tau)u(t-\tau)d\tau$$

=
$$\int_{0}^{\infty} 1u(t-\tau)d\tau$$

=
$$\int_{0}^{t} 1d\tau = \begin{cases} t & t \ge 0\\ 0 & t < 0 \end{cases}$$
 (9.39)

The upper limit becomes t because $u(t - \tau) = 1$ when $t - \tau \ge 0$, or $\tau \le t$. The final answer (9.39) can be written concisely as

$$u(t) * u(t) = t u(t)$$
(9.40)

The signal tu(t) is called a *unit ramp* because it is linearly increasing with a slope of one. This example illustrates that one skill in doing a convolution integral is the proper manipulation of the limits of integration prior to performing the actual integration.

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