EXERCISE 11.1: Confirm that (11.16) is a valid Fourier transform pair even when a is a complex number, as long as $\Re e\{a\} > 0$.

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SOLUTION

$$x(t) = e^{-at}u(t) \xrightarrow{\text{F.T.}} X(j\omega) = \frac{1}{a+j\omega}$$

Let $a = \alpha + j\beta$ where $\alpha = \Re\{a\}$.

In order to have a Fourier transform the signal must be absolutely integrable, i.e., $\int_{-\infty}^{\infty} |x(t)|\,dt < \infty$

If we apply this test:

$$\int_{-\infty}^{\infty} |x(t)|\,dt = \int_{-\infty}^{\infty} |e^{-at}u(t)|\,dt$$

$$e^{-at} = e^{-(\alpha+j\beta)t} = e^{-\alpha t}e^{-j\beta t}$$

$$= \int_{0}^{\infty} |e^{-\alpha t}e^{-j\beta t}|\,dt$$

$$|e^{-j\beta t}| = 1$$

$$= \int_{0}^{\infty} |e^{-\alpha t}|\,|e^{-j\beta t}|\,dt$$

$$= \int_{0}^{\infty} e^{-\alpha t}\,dt = \left.\frac{e^{-\alpha t}}{-\alpha}\right|_{0}^{\infty} = \frac{\lim_{t\to\infty} e^{-\alpha t} - 1}{-\alpha}$$

We need $\lim_{t\to\infty} e^{-at}$ to be finite. If $\alpha > 0$, the limit is zero.

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