EXERCISE 11.10: Use convolution to find the time function that corresponds to the Fourier transform

$$Y(j\omega) = \left(\frac{\sin(2\omega)}{(\omega/2)}\right)\left(\frac{\sin(\omega)}{(\omega/2)}\right)$$

McClellan, Schafer and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.

SOLUTION

$$Y(j\omega) = \left(\frac{\sin 2\omega}{\omega/2}\right)\left(\frac{\sin \omega}{\omega/2}\right)$$

pulse of length 4, $u(t+2) - u(t-2)$

inverse transform is a pulse of length 2, extending from $t=-1$ to $t=+1$: $u(t+1) - u(t-1)$

$$y(t) = [u(t+2) - u(t-2)] * [u(t+1) - u(t-1)]$$

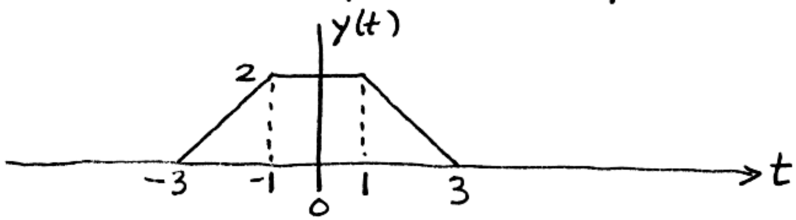
$$= u(t+2) * u(t+1) - u(t+2) * u(t-1) - u(t-2) * u(t+1) + u(t-2) * u(t-1)$$

$$= (t+3)u(t+3) - (t+1)u(t+1) - (t-1)u(t-1) + (t-3)u(t-3)$$

(starts at $t=-3$; at $t=-1$; at $t=1$; at $t=3$)

$$= \begin{cases} 0 & \text{for } t < -3 \\ (t+3) & \text{for } -3 \le t \le -1 \\ 2 & \text{for } -1 \le t \le 1 \\ (-t+3) & \text{for } 1 \le t \le 3 \\ 0 & \text{for } t > 3 \end{cases}$$

The plot has the shape of a trapezoid:



There are many ways to perform this convolution, but here we used a method based on convolving unit steps:

$$u(t-t_1) * u(t-t_2) = (t-t_1-t_2)u(t)$$

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