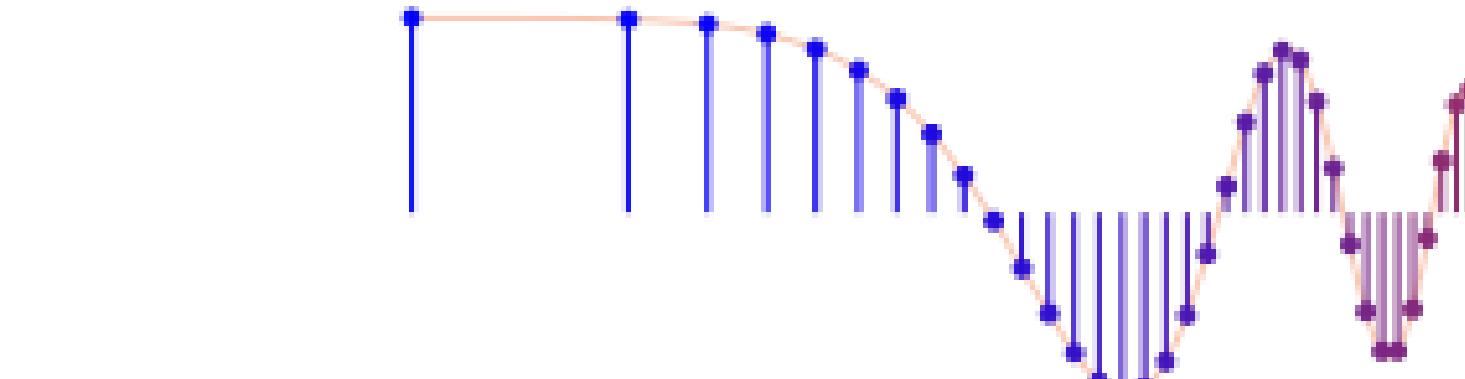


EXERCISE 11.11: Prove that the modulator system in Fig. 11-16 is linear, but not time-invariant.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458. © 2003 Pearson Education, Inc.



SOLUTION



Modulator system is: $y(t) = p(t)x(t)$

Linearity: Let $y_1(t) = p(t)x_1(t)$ and $y_2(t) = p(t)x_2(t)$

$$\text{Let } x(t) = \alpha x_1(t) + \beta x_2(t)$$

$$\text{Then } y(t) = p(t)x(t)$$

$$\begin{aligned} &= p(t)[\alpha x_1(t) + \beta x_2(t)] \\ &= \alpha p(t)x_1(t) + \beta p(t)x_2(t) \\ &= \alpha y_1(t) + \beta y_2(t) \end{aligned}$$

Thus, the modulator system is linear

Time-Invariance:

If $p(t)$ is constant, the system is time-invariant

If $p(t)$ is not constant, then there are two times t_1 and t_2 such that $p(t_1) \neq p(t_2)$.

$$\text{Let } x(t) = \delta(t-t_1) \delta(t-t_1)$$

$$\begin{aligned} \text{Then the output is } y(t) &= p(t) \delta(t-t_1) \\ &= p(t_1) \delta(t-t_1) \end{aligned}$$

Now shift the input by (t_2-t_1)

$$x(t-(t_2-t_1)) = \delta(t-(t_2-t_1)-t_1) = \delta(t-t_2)$$

$$\begin{aligned} \text{Then the output is } y_2(t) &= p(t) \delta(t-t_2) \\ &= p(t_2) \delta(t-t_2) \end{aligned}$$

However, if we shift $y(t)$ by (t_2-t_1) , we get

$$y(t-(t_2-t_1)) = p(t_1) \delta(t-(t_2-t_1)-t_1)$$

$$= p(t_1) \delta(t-t_2)$$

These are different, so the system is not time-invariant