EXERCISE 11.11: Prove that the modulator system in Fig. 11-16 is linear, but not time-invariant.

McClellan, Schafer and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.

SOLUTION

Modulator system is: $y(t) = p(t)x(t)$

<u>Linearity</u>: Let $y_1(t) = p(t)x_1(t)$ and $y_2(t) = p(t)x_2(t)$

Let $x(t) = \alpha x_1(t) + \beta x_2(t)$

Then
$$\begin{aligned} y(t) &= p(t)x(t) \\ &= p(t)[\alpha x_1(t) + \beta x_2(t)] \\ &= \alpha\, p(t)x_1(t) + \beta\, p(t)x_2(t) \\ &= \alpha\, y_1(t) + \beta\, y_2(t) \end{aligned}$$

Thus, the modulator system is <u>linear</u>

<u>Time-Invariance:</u>

If $p(t)$ is constant, the system is time-invariant

If $p(t)$ is not constant, then there are two times t_1 and t_2 such that $p(t_1) \neq p(t_2)$.

Let $x(t) = \delta(t-t_1)$

Then the output is
$$\begin{aligned} y(t) &= p(t)\,\delta(t-t_1) \\ &= p(t_1)\,\delta(t-t_1) \end{aligned}$$

Now shift the input by $(t_2 - t_1)$

$$x(t-(t_2-t_1)) = \delta(t-(t_2-t_1)-t_1) = \delta(t-t_2)$$

Then the output is
$$\begin{aligned} y_2(t) &= p(t)\,\delta(t-t_2) \\ &= p(t_2)\,\delta(t-t_2) \end{aligned}$$

However, if we shift $y(t)$ by $(t_2 - t_1)$, we get
$$\begin{aligned} y(t-(t_2-t_1)) &= p(t_1)\,\delta(t-(t_2-t_1)-t_1) \\ &= p(t_1)\,\delta(t-t_2) \end{aligned}$$

These are different, so the system is <u>not</u> time-invariant