EXERCISE 11.2:

The signal $x(t) = e^{bt}u(-t)$ is an example of a *left-sided real exponential signal*. Sketch this signal for $b > 0$ and then show that the Fourier transform of this signal is

$$X(j\omega) = \frac{1}{b - j\omega}$$

if $b > 0$. Also, using (11.15), show that the Fourier transform does not exist if $b \leq 0$.

McClellan, Schafer and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

$x(t) = e^{bt}u(-t)$ is <u>LEFT</u> sided.

If $b>0$, then $\lim\limits_{t\to-\infty} e^{bt} = 0$, also $\lim\limits_{t\to-\infty} e^{bt}e^{-j\omega t} = 0$.

the term $e^{-j\omega t}$ is bounded, so $e^{bt} \to 0$ controls the limit

$$X(j\omega) = \int_{-\infty}^{\infty} e^{bt}u(-t)\,e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{0} e^{(b-j\omega)t}\,dt$$

$$= \left.\frac{e^{(b-j\omega)t}}{(b-j\omega)}\right|_{-\infty}^{0} = \frac{1-\lim\limits_{t\to-\infty} e^{(b-j\omega)t}}{b-j\omega} = \frac{1}{b-j\omega}$$

If $b \le 0$, then $\lim\limits_{t\to-\infty} e^{bt}e^{-j\omega t}$ does not exist.

If $b<0$, $\lim\limits_{t\to-\infty} e^{bt} \to \infty$; if $b=0$, the term $e^{-j\omega t}$ oscillates so there is no limit.