EXERCISE 11.8: For the real odd function $x(t) = e^{at}u(-t) - e^{-at}u(t)$, show that the Fourier transform is

$$X(j\omega) = \frac{2j\omega}{a^2 + \omega^2}$$

which is an imaginary odd function of ω.

McClellan, Schafer and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

$$x(t) = e^{at}u(-t) - e^{-at}u(t)$$

Use linearity to do the transforms of each part separately.

$$e^{-at}u(t) \longrightarrow \frac{1}{a+j\omega}$$

$$e^{at}u(-t) \longrightarrow \frac{1}{a-j\omega}$$

Thus, $X(j\omega) = \dfrac{1}{a-j\omega} - \dfrac{1}{a+j\omega}$

$$= \frac{a+j\omega-(a-j\omega)}{(a-j\omega)(a+j\omega)}$$

$$= \frac{2j\omega}{a^2+\omega^2}$$

Note: $x(t)$ is odd because $x(-t) = -x(t)$.

In addition $X(j\omega)$ is odd:

$$X(-j\omega) = \frac{2j(-\omega)}{a^2+(-\omega)^2} = -\frac{2j\omega}{a^2+\omega^2} = -X(j\omega)$$

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