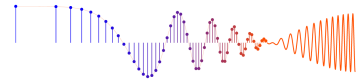


EXERCISE 11.8: For the real odd function $x(t) = e^{at}u(-t) - e^{-at}u(t)$, show that the Fourier transform is

$$X(j\omega) = \frac{2j\omega}{a^2 + \omega^2}$$

which is an imaginary odd function of ω .



$$x(t) = e^{at}u(-t) - e^{-at}u(t)$$

Use linearity to do the transforms of each part separately.

$$e^{-at}u(t) \longrightarrow \frac{1}{a+j\omega}$$

$$e^{at}u(-t) \longrightarrow \frac{1}{a-j\omega}$$

$$\begin{aligned}\text{Thus, } \underline{X}(j\omega) &= \frac{1}{a-j\omega} - \frac{1}{a+j\omega} \\ &= \frac{a+j\omega - (a-j\omega)}{(a-j\omega)(a+j\omega)} \\ &= \frac{2j\omega}{a^2 + \omega^2}\end{aligned}$$

Note: $x(t)$ is odd because $x(-t) = -x(t)$.

In addition $\underline{X}(j\omega)$ is odd:

$$\underline{X}(-j\omega) = \frac{2j(-\omega)}{a^2 + (-\omega)^2} = -\frac{2j\omega}{a^2 + \omega^2} = -\underline{X}(j\omega)$$