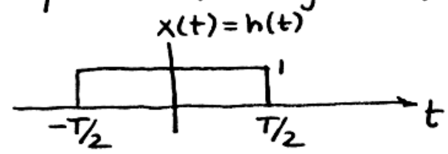
EXERCISE 11.9: Show by evaluating the convolution of two identical pulses, each one defined by (11.67), that $y(t)$ is the triangle function shown in Fig. 11-15(a).

McClellan, Schafer and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

$x(t)$ and $h(t)$ are both pulses of length T, extending from $-T/2$ to $+T/2$.



There are many ways to perform this convolution, but here we'll use a method based on convolving unit steps:

$$u(t-t_1) * u(t-t_2) = (t-t_1-t_2)u(t)$$

The formula above is easy to verify by plugging into the convolution integral:

$$u(t-t_1) * u(t-t_2) = \int_{-\infty}^{\infty} u(\tau-t_1)\,u(t-\tau-t_2)\,d\tau$$

$$= \int_{t_1}^{t-t_2} 1\,d\tau \quad \leftarrow \text{if } t-t_2 \geq t_1 \text{ or } t \geq t_1+t_2$$

$$= t-t_2-t_1$$

Note: if $t < t_1+t_2$ the integral is zero

Now, $x(t) = h(t) = u(t+T/2) - u(t-T/2)$

Thus

$$x(t) * h(t) = \left[u(t+T/2) - u(t-T/2)\right] * \left[u(t+T/2) - u(t-T/2)\right]$$

$$= u(t+T/2)*u(t+T/2) - u(t-T/2)*u(t+T/2) - u(t+T/2)*u(t-T/2) + u(t-T/2)*u(t-T/2)$$

$$= (t+T)u(t+T) - 2t\,u(t) + (t-T)u(t-T)$$

($(t+T)u(t+T)$ starts at $t=-T$; $2t\,u(t)$ starts at $t=0$; $(t-T)u(t-T)$ starts at $t=T$)

$$= \begin{cases} t+T & \text{for } -T \leq t \leq 0 \\ \underbrace{(t+T)-2t}_{T-t} & \text{for } 0 \leq t \leq T \\ \underbrace{(t+T)-2t+(t-T)}_{0} & \text{for } t \geq T \end{cases}$$