



**EXERCISE 9.3:** The differentiator system is defined by the input/output relation

$$y(t) = \frac{dx(t)}{dt}$$

Show that the differentiator system is both linear and time-invariant (LTI).



$$y(t) = \frac{d}{dt} x(t)$$

Linearity: let  $w(t) = \alpha x_1(t) + \beta x_2(t)$

$$\begin{aligned} y_w(t) &= \frac{d}{dt} \{ \alpha x_1(t) + \beta x_2(t) \} = \alpha \frac{d}{dt} x_1(t) + \beta \frac{d}{dt} x_2(t) \\ &= \alpha y_1(t) + \beta y_2(t) \end{aligned}$$

$$\text{where } y_1(t) = \frac{d}{dt} x_1(t) \text{ and } y_2(t) = \frac{d}{dt} x_2(t)$$

Time-Invariance:

$$\text{Let } w(t) = x(t - t_1). \text{ Then } y_w(t) = \frac{d}{dt} x(t - t_1)$$

Compare  $y_w(t)$  to  $y(t - t_1)$ :

$$y(t - t_1) = \frac{d}{dt} x(t - t_1)$$

Since they are the same, the system is  
Time-invariant