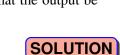
EXERCISE 9.5: Convince yourself of the truth of the following statement: If the input contains no impulses, but is discontinuous (like u(t)), then the impulse response must contain at least one impulse in order that the output be

discontinuous too.



McClellan, Schafer and Yoder, Signal Processing First, ISBN 0-13-065562-7. Prentice Hall, Upper Saddle River, NJ 07458. © 2003 Pearson Education, Inc.



The point of this problem is that convolution will perform "smoothing," so the output will be smoother than the input.

Note: in order that this statement be true, x(t) cannot contain any impulses. The discontinuity in x(t) has to be a "jump."

Proof:

Assume y(t) has a jump at $t=t_1$. If we take the derivative of y(t), we get ______ other stuff

$$\frac{d}{dt}y(t) = A\delta(t-t_i) + \hat{y}(t)$$

$$t_{\text{size of jump}}$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau \implies \frac{d}{d\tau} y(t) = \int_{-\infty}^{\infty} h(\tau) \frac{d}{d\tau} x(t-\tau) d\tau$$

In other words,
$$\frac{d}{dt}y(t) = h(t) * \frac{d}{dt}x(t)$$

Since x(t) is discontinuous, its derivative has at least one impulse: $\frac{d}{dt}x(t) = B\delta(t-t_2) + \hat{x}(t)$ Lsize of jump at $t=t_2$.

The only way to have equality is for h(t) to have one impulse, such as $\frac{A}{B}\delta(t-t_1+t_2)$