EXERCISE 9.5: Convince yourself of the truth of the following statement: If the input contains no impulses, but is discontinuous (like $u(t)$), then the impulse response must contain at least one impulse in order that the output be discontinuous too.

McClellan, Schafer and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

The point of this problem is that convolution will perform "smoothing," so the output will be smoother than the input.

Note: in order that this statement be true, $x(t)$ cannot contain any impulses. The discontinuity in $x(t)$ has to be a "jump."

<u>Proof</u>:

Assume $y(t)$ has a jump at $t = t_1$. If we take the derivative of $y(t)$, we get

$$\frac{d}{dt} y(t) = A\delta(t - t_1) + \hat{y}(t)$$

(A ← size of jump; $\hat{y}(t)$ ← other stuff)

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \;\Rightarrow\; \frac{d}{dt} y(t) = \int_{-\infty}^{\infty} h(\tau)\frac{d}{dt}x(t-\tau)\,d\tau$$

In other words, $\frac{d}{dt} y(t) = h(t) * \frac{d}{dt} x(t)$

Since $x(t)$ is discontinuous, its derivative has at least one impulse: $\frac{d}{dt} x(t) = B\delta(t - t_2) + \hat{x}(t)$

(B ← size of jump at $t = t_2$.)

Thus

$$A\delta(t - t_1) + \hat{y}(t) = h(t) * B\delta(t - t_2) + h(t) * \hat{x}(t)$$

The only way to have equality is for $h(t)$ to have one impulse, such as $\frac{A}{B}\delta(t - t_1 + t_2)$

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