EXERCISE 9.7: Repeat the convolution of $x(t) = u(t-1)$ with $h(t) = e^{-t}u(t)$, but this time use the convolution integral form

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Because convolution is commutative, your answer should be the same as (9.52).

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SOLUTION

$$x(t) = u(t-1) \quad \text{and} \quad h(t) = e^{-t}u(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = \int_{-\infty}^{\infty} u(\tau-1)\,e^{-(t-\tau)}u(t-\tau)\,d\tau$$

$$= \int_{1}^{t} e^{-t}e^{\tau}\,d\tau \quad \text{if } t \geq 1$$

Note: integral is zero if $t<1$

$$= e^{-t}e^{\tau}\Big|_{1}^{t} = e^{-t}(e^{t} - e)$$

$$\Rightarrow y(t) = \left(1 - e^{-t+1}\right)u(t-1)$$

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