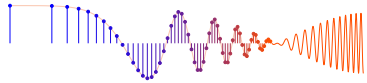


PROBLEM:

In each of the following problems, find the Fourier transform, or inverse Fourier transform. Give your answer as a simple formula or a plot. **Explain** each answer by stating which property and transform pair you used.

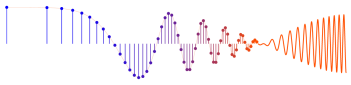
(a) Find $X(j\omega)$ when $x(t) = 3\delta(t-1) + e^{-2t}u(t-1)$.

(b) Find $h(t)$ when $H(j\omega) = \dfrac{j\omega}{4+3j\omega}$.

(c) Find $V(j\omega)$ when $v(t) = \begin{cases} 1 & 3 \le t < 7 \\ 0 & \text{otherwise} \end{cases}$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION



In each of the following problems, find the Fourier transform, or inverse Fourier transform. Give your answer as a simple formula or a plot. **Explain** each answer by stating which property and transform pair you used.

(a) Find $X(j\omega)$ when $x(t) = 3\delta(t-1) + e^{-2t}u(t-1)$.

$$\delta(t) \longleftrightarrow 1 \quad \text{and} \quad e^{-at}u(t) \longleftrightarrow \frac{1}{1+a\,j\omega}$$

Using linearity and the delay property we have:

$$x(t) = 3\,\delta(t-1) + e^{-2}e^{-2(t-1)}u(t-1)$$

$$X(j\omega) = 3e^{-j\omega} + e^{-2}e^{-j\omega}\,\frac{1}{2+j\omega},$$

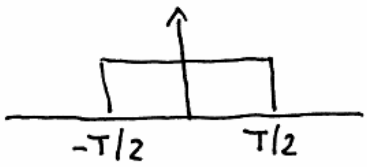
(b) Find $h(t)$ when $H(j\omega) = \dfrac{j\omega}{4+3j\omega}$.

$$H(j\omega) = \frac{j\omega}{3}\cdot\frac{1}{\frac{4}{3}+j\omega} \qquad \text{(use the derivative prop)}$$

$$h(t) = \frac{d}{dt}\left\{\frac{1}{3}e^{-\frac{4}{3}t}u(t)\right\} = -\frac{4}{9}e^{-\frac{4}{3}t}u(t) + \frac{1}{3}\cancel{e^{0}}\,\delta(t)$$

(c) Find $V(j\omega)$ when $v(t) = \begin{cases} 1 & 3 \le t < 7 \\ 0 & \text{otherwise} \end{cases}$.

Recall



$$\longleftrightarrow \quad \frac{\sin(\omega T/2)}{\omega/2}$$

Using the delay property, and noting that $T=4$ we have

$$V(j\omega) = e^{-j5\omega}\cdot\frac{\sin(2\omega)}{\omega/2}$$