PROBLEM:

Evaluate the following and give the answer in both rectangular and polar form. In all cases, assume that the complex numbers are $z_1 = -2 - j2$ and $z_2 = 3e^{-j3\pi/4}$.

(a) Conjugate: z_1^*	(d) z_2^2	(g) $z_1 + z_2^*$
(b) jz_2	(e) $z_1^{-1} = 1/z_1$	(h) $\|z_2\|^2 = z_2 z_2^*$
(c) z_2/z_1	(f) $z_1 z_2$	(i) $z_2 + z_2^*$

Note: z^* means the "conjugate" of z. Part (h) is the *magnitude-squared*.

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SOLUTION

(a) $z_1 = -2 - j2 \qquad z_1^* = -2 - (-j2) = -2 + j2$

(b) $jz_2 = j\left(3e^{-j\frac{3\pi}{4}}\right) = e^{j\pi/2} \cdot 3e^{-j\frac{3\pi}{4}} = 3e^{j\left(\frac{\pi}{2} - \frac{3\pi}{4}\right)}$

$= 3e^{-j\frac{\pi}{4}}$ or $2.12 - j2.12$

(c) $\dfrac{z_2}{z_1} = \dfrac{3e^{-j\frac{3\pi}{4}}}{-2-j2} = \dfrac{3e^{-j\frac{3\pi}{4}}}{2\sqrt{2}\,e^{j\frac{5\pi}{4}}} = 1.06e^{-j\left(\frac{3\pi}{4} + \frac{5\pi}{4}\right)}$

$= 1.06e^{-j\frac{8\pi}{4}} = 1.06e^{-j2\pi} = 1.06e^{j0} = 1.06 + j0$

(d) $z_2^2 = \left(3e^{-j3\pi/4}\right)^2 = 9e^{-j\frac{3\pi}{2}} = 0 - 9j$

(e) $z_1^{-1} = \dfrac{1}{-2-j2} = \dfrac{-2+j2}{(-2-j2)(-2+j2)} = \dfrac{-2+j2}{4+4}$

$= -\frac{1}{4} + j\frac{1}{4} = \frac{\sqrt{2}}{4}e^{j\frac{3\pi}{4}}$

(f) $z_1 z_2 = (-2-j2)3e^{-j\frac{3\pi}{4}} = 2\sqrt{2}\,e^{j\frac{5\pi}{4}} \cdot 3e^{-j\frac{3\pi}{4}}$

$= 6\sqrt{2}\,e^{j\frac{2\pi}{4}} = 6\sqrt{2}\,e^{j\frac{\pi}{2}} = 0 + j6\sqrt{2}$

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$$(g)\ z_1 + z_2^* = -2+j2 + 3e^{+j\frac{3\pi}{4}} =$$

$$= -2+j2 + 3\left(\cos\frac{3\pi}{4} + j\sin\frac{3\pi}{4}\right)$$

$$= -2+j2 + 3\left(-\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) = -2+j2 - \frac{3}{2}\sqrt{2} + j\frac{3\sqrt{2}}{2}$$

$$= -4.12 + j4.12 = \sqrt{2}(4.12)e^{j\frac{3\pi}{4}}$$

$$(h)\ |z_2^2| = z_2 z_2^* = 3e^{-j\frac{3\pi}{4}} \cdot 3e^{+j\frac{3\pi}{4}} = 9$$

$$(i)\ z_2 + z_2^* = 2\,\mathcal{Re}[z_2] = 2\left(3\cos\frac{3\pi}{4}\right)$$

$$= 6\frac{\sqrt{2}}{2} = 3\sqrt{2}$$

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