PROBLEM:

Simplify the following complex-valued expressions. Give your answer in either rectangular or polar form, whichever is most convenient. In parts (a)-(d) assume that A, α, and ϕ are positive real numbers. Your answers to parts (a)-(d) will be in terms of these quantities.

(a) For $z = Ae^{-j\pi/3}$, determine a simple expression for $\Im m\{z^*\}$.

(b) For $z = Ae^{-j\pi/3}$, determine a simple expression for $z + z^*$.

(c) For $z = 10e^{j\phi}$, determine a simple expression for $\Re e\{jz\}$.

(d) For $z = -\alpha + j\alpha$, determine a simple expression for z in polar form.

McClellan, Schafer and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.

SOLUTION

(a) $z = Ae^{-j\pi/3} = A\left(\cos\left(\frac{-\pi}{3}\right) + j\sin\left(-\frac{\pi}{3}\right)\right)$

$z^* = A\left(\cos\left(-\frac{\pi}{3}\right) - j\sin\left(-\frac{\pi}{3}\right)\right)$

$= A\left(\cos\left(-\frac{\pi}{3}\right) + j\sin\left(\frac{\pi}{3}\right)\right)$

$\mathrm{Im}[z^*] = A\sin\left(\frac{\pi}{3}\right) = A\frac{\sqrt{3}}{2}$

(b) $z = Ae^{-j\pi/3}$

$z + z^* = Ae^{-j\pi/3} + Ae^{+j\pi/3}$

but $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

so $z + z^* = A\left(e^{j\pi/3} + e^{-j\pi/3}\right) = 2A\cos\pi/3$

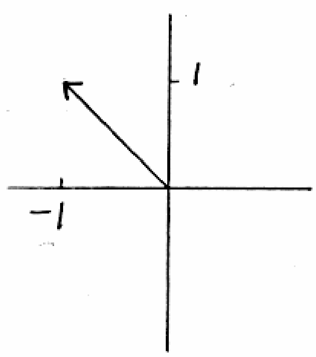
$= 2A\left(\frac{1}{2}\right) = A$

(c) $z = 10e^{j\phi} = 10(\cos\phi + j\sin\phi)$

$\mathrm{Re}[jz] = -10\sin\phi$

(d) $z = -\alpha + j\alpha = \alpha(-1 + j)$

$z = \sqrt{2}\alpha e^{j\frac{3\pi}{4}}$



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