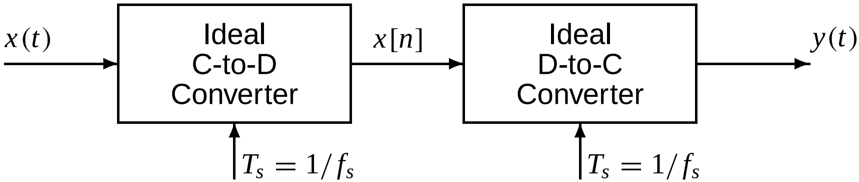
PROBLEM:



Shown in the figure above is an ideal C-to-D converter that samples $x(t)$ with a sampling period T_s to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$. Suppose that $x(t)$ is given by

$$x(t) = [15 + 30\sin(250\pi t)]\cos(1000\pi t)$$

(a) Sketch the two-sided spectrum of this signal. Be sure to label important features of the plot. *Hint: Recall the AM spectrum from a previous homework set.*

(b) Is this waveform periodic? If so, what is the period?

(c) What is the minimum sampling rate f_s that can be used in the above system so that $y(t) = x(t)$?

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

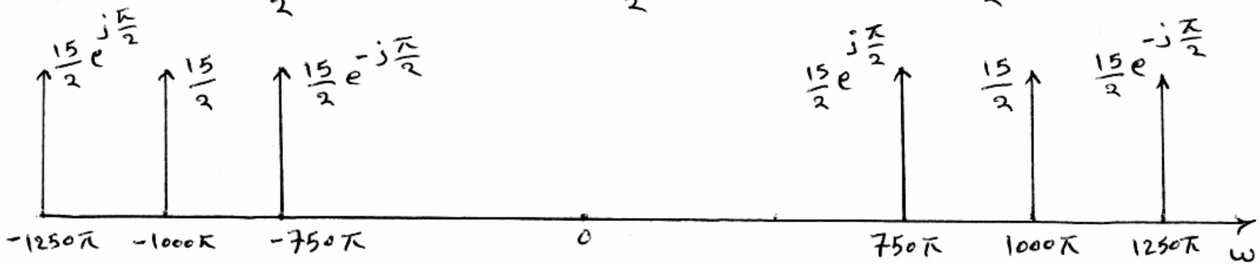
SOLUTION

(a) $x(t) = 15\cos(1000\pi t) + 30\sin(250\pi t)\cos(1000\pi t)$

$$\rightarrow x(t) = 15\cos(1000\pi t) + 30\left[\frac{1}{2j}e^{j250\pi t} - \frac{1}{2j}e^{-j250\pi t}\right]\cdot\left[\frac{1}{2}e^{j1000\pi t} + \frac{1}{2}e^{-j1000\pi t}\right]$$

$$\rightarrow x(t) = \overbrace{\frac{15}{2}e^{j1000\pi t} + \frac{15}{2}e^{-j1000\pi t}} + \frac{15}{2}e^{-j\frac{\pi}{2}}e^{j1250\pi t} +$$

$$+ \frac{15}{2}e^{j\frac{\pi}{2}}e^{-j1250\pi t} + \frac{15}{2}e^{j\frac{\pi}{2}}e^{j750\pi t} + \frac{15}{2}e^{-j\frac{\pi}{2}}e^{-j750\pi t}$$



(b) Since $\underbrace{\omega_0 = 250\pi}_{\text{is the largest number which}}$ divides 750π, 1000π, 1250π,

we conclude that $\omega_0 = 250\pi$ is the fundamental frequency (rad/s). Thus the signal is periodic with period $T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{250\pi} = 8$ msec

(c) $f_s \geq 2f_{max}$ $\qquad f_{max} = \frac{1250\pi}{2\pi}$

$\rightarrow f_s \geq 1250$ HZ $\qquad$ Thus $(f_s)_{min} = 1250$ HZ

McClellan, Schafer, and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.