PROBLEM:

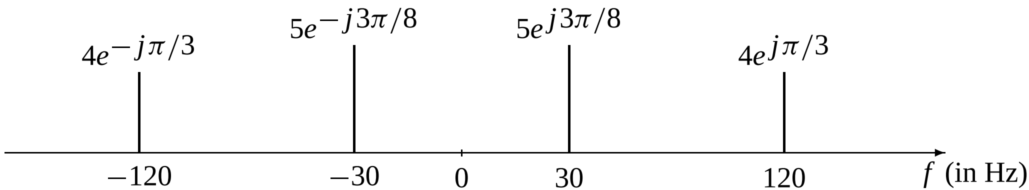
Again consider the ideal C-to-D converter and ideal D-to-C converter shown in previous problem.

(a) Suppose that a discrete-time signal $x[n]$ is given by the formula

$$x[n] = 4\cos(0.125\pi n + \pi/8)$$

If the sampling rate of the C-to-D converter is $f_s = 2000$ samples/second, many *different* continuous-time signals $x(t) = x_\ell(t)$ could have been inputs to the above system. Determine two such inputs with frequency less than 2000 Hz; i.e., find $x_1(t)$ and $x_2(t)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 1/2000$ secs.

(b) Now if the input $x(t)$ is given by the two-sided spectrum representation shown below,



Determine the spectrum for $x[n]$ when $f_s = 120$ samples/sec. Make a $\boxed{\text{plot}}$ for your answer, but label the frequency, amplitude and phase of each spectral component.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

Since $x[n]$ is given in sinusoids form, we suggest

$x_1(t) = A_1 \cos(2\pi f_1 t + \varphi_1)$ in which the parameters

A_1, f_1, and φ_1 have to be found.

we use $x[n] = x_1(nT_s)$ where $T_s = \frac{1}{2000}$

$$\rightarrow 4\cos(0.125\pi n + \frac{\pi}{8}) = A_1 \cos(2\pi f_1 \frac{n}{2000} + \varphi_1)$$

$$\Rightarrow A_1 = 4 \quad , \quad \varphi_1 = \frac{\pi}{8} \quad , \quad f_1 = 125 \text{ Hz}$$

Therefore, $x_1(t) = 4\cos(2\pi(125)t + \frac{\pi}{8})$

To find $x_2(t)$ (that would give the same $x[n]$),

we use the fact that adding $2\pi k$ $(k = \pm 1, \pm 2, \ldots)$

to $\hat{\omega}$ (the frequency of $x[n]$) does not change

anything. i.e., $x[n] = 4\cos((0.125\pi + 2k\pi)n + \frac{\pi}{8})$

If we start with $x_2(t) = A_2 \cos(2\pi f_2 t + \varphi_2)$,

then $x_2(t)$ can be found from $x_2(nT_s) = x[n]$

Following the same approach we had for $x_1(t)$,

we obtain:

$$A_2 = 4 \quad , \quad \varphi_2 = \frac{\pi}{8} \quad ,$$

$$\frac{2\pi f_2}{2000} = 0.125\pi + 2K\pi \qquad K = \pm 1, \pm 2, \ldots$$

$$\rightarrow f_2 = 125 + 2000K \qquad K = \pm 1, \pm 2, \ldots$$

Since we require $f_2 < 2000$, thus we choose

$$K = -1 \quad \rightarrow \quad f_2 = -1875$$

$$\rightarrow x_2(t) = 4\cos\left(-2\pi(1875)t + \frac{\pi}{8}\right)$$

$$= 4\cos\left(2\pi(1875)t - \frac{\pi}{8}\right)$$

because $\cos(-\theta) = \cos(\theta)$ for any θ

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(b) First we find $x(t)$ from the spectrum:

$$x(t) = 10\cos\left(2\pi(30)t + \frac{3\pi}{8}\right) + 8\cos\left(2\pi(120)t + \frac{\pi}{3}\right)$$

Now, we obtain $x[n]$. Since $f_s = 120$ samples/sec we expect that there is an aliasing term introduced by the second cosine term in $x(t)$.

$x[n] = x(nT_s)$ where $T_s = \frac{1}{120}$

$$x[n] = 10\cos\left(\frac{\pi}{2}n + \frac{3\pi}{8}\right) + 8\cos\left(2\pi n + \frac{\pi}{3}\right)$$

Thus $x[n]$ has two frequency components:

$\hat{\omega}_1 = \frac{\pi}{2}$, $\hat{\omega}_2 = 2\pi$

To plot the spectrum of $x[n]$, we treat it similar to the plot of the spectrum for continuous-time signals. But we need to keep in mind two things:

(1) We plot the spectrum of any arbitrary $x[n]$ in the interval $-\pi \leq \hat{\omega} \leq \pi$.

(2) For those frequency components that are not in the interval $-\pi \leq \omega \leq \pi$, we subtract (or add) multiples of (2π) such

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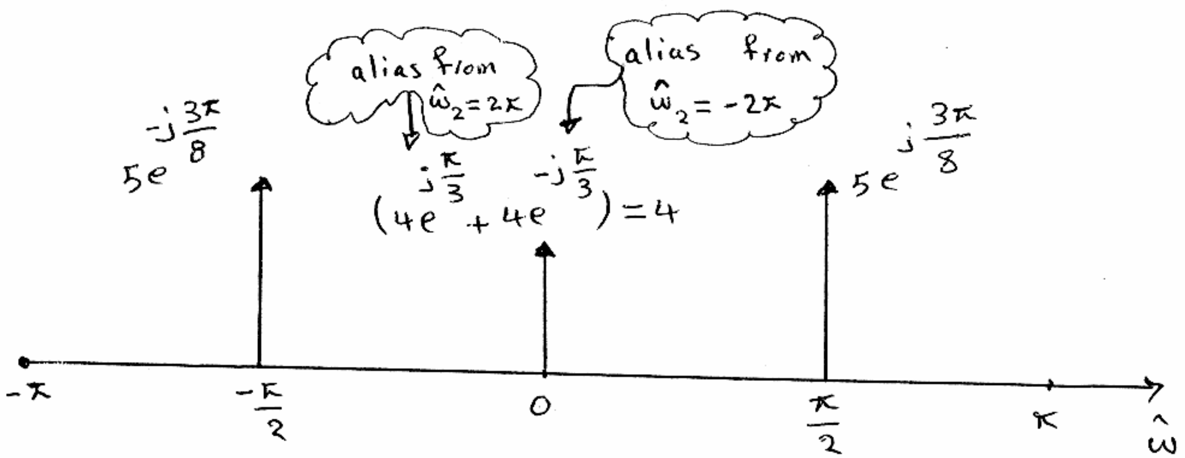
that the new frequency lies in the interval

$-\pi \le \hat{\omega} \le \pi$.

Note that step (2) in the above is required for all aliasing terms.

Now, since $\hat{\omega}_1 = \frac{\pi}{2}$, step (2) is not required.

But for $\hat{\omega}_2 = 2\pi$, we follow step (2). This gives a new $\hat{\omega}_2 = 2\pi - 2\pi = 0$



Note to the DC term introduced by aliasing frequecies.