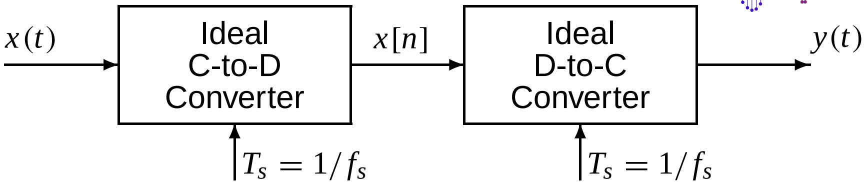
PROBLEM:



Chirps are very useful signals for probing the behavior of sampling operations and illustrating the "folding" type of aliasing.

(a) If the input to the ideal C/D converter is $x(t) = 7\cos(1800\pi t + \pi/4)$, and the sampling frequency is 1000 Hz, then the output $y(t)$ is a sinusoid. Determine the formula for the output signal.

(b) Suppose that the input signal is a chirp signal defined as follows:

$$x(t) = \cos(2000\pi t - 400\pi t^2) \qquad \text{for } 0 \le t \le 5 \text{ sec.}$$

If the sampling rate is $f_s = 1000$ Hz, then the output signal $y(t)$ will have time-varying frequency content. Draw a graph of the resulting analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction.** Hint: this could be done in MATLAB by putting a sampled chirp signal into the MATLAB function `specgram()`, or the SP-First function `plotspec()`.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
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SOLUTION

(a)

$$
\begin{aligned}
f_s &= 1000 \text{ Hz} \\
x[n] &= x(n/f_s) \\
&= 7\cos(1800\pi \cdot n/1000 + \pi/4) \\
&= 7\cos(1.8\pi n + \pi/4) \\
&= 7\cos(1.8\pi n - 2\pi n + \pi/4) \\
&= 7\cos(-0.2\pi n + \pi/4) \\
&= 7\cos(0.2\pi n - \pi/4) \\
y(t) &= x[t f_s] \\
&= 7\cos(0.2\pi \cdot t \cdot 1000 - \pi/4) \\
&= 7\cos(200\pi t - \pi/4)
\end{aligned}
$$

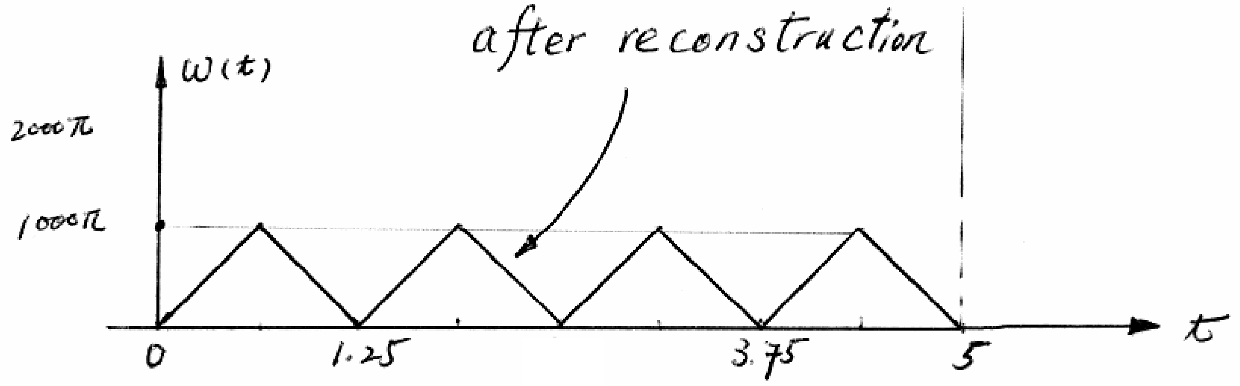
(b)

$\cos(2000\pi t - 400\pi t^2)$

$\Rightarrow \psi(t) = 2000\pi t - 400\pi t^2$

$\omega_i(t) = \frac{d}{dt}\psi(t) = 2000\pi - 800\pi t$

With a sampling frequency of $f_s = 1000$ Hz, the highest frequency reconstructed by the D / A converter would be $\omega = 2\pi(500)$ rad / sec. For example at $t = 4$, $\omega_i(4) = 2000\pi - 3200\pi = -1200\pi$ rad / sec. which aliases to $\omega = 2\pi(-1200+1000) = -2\pi(200)$ rad / sec. Since a cosine with negative frequency is the same as positive frequency, the output frequency is $2\pi(200)$ rad / sec.



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