PROBLEM:

Try your hand at expressing each of the following in a simpler form:

(a) Convolution: $\delta(t-1) * [\delta(t+2) + 2e^{-t+1}\sin(\pi t)u(t+1)] =$

(b) Multiplication: $[u(-t+3) - u(t)][\delta(t+1) + \delta(t+4)] =$

(c) $\dfrac{d}{dt}\left[\sin(5\pi t)u(t-\tfrac{1}{2})\right] =$

(d) $\displaystyle\int_{-\infty}^{t} e^{-2\tau-1}\delta(\tau-3)d\tau =$

Note: use properties of the impulse signal $\delta(t)$ and the unit-step signal $u(t)$ to perform the simplifications. For example, recall

$$\delta(t) = \frac{d}{dt}u(t) \qquad \text{where} \qquad u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Be careful to distinguish between multiplication and convolution; convolution is denoted by a "star", as in $u(t) * \delta(t-2)$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) "Star-ing" with $\delta(t-1)$ means delaying by one unit

So: $\delta(t+1) + 2e^{-(t-1)+1}\sin(\pi(t-1))\,u((t-1)+1)$

$= \delta(t+1) - 2e^{-t+2}\sin(\pi t)\,u(t)$

(b) $[u(4) - u(-1)]\delta(t+1) + [u(7) - u(-4)]\,\delta(t+4) = \delta(t+1) + \delta(t+4)$

(c) Use product rule

$$\frac{d}{dt}\sin(5\pi t)\,u(t-\tfrac{1}{2}) = 5\pi\cos\pi t \cdot u(t-\tfrac{1}{2}) + \sin(5\pi t)\,\delta(t-\tfrac{1}{2})$$

$$= 5\pi\cos\pi t \cdot u(t-\tfrac{1}{2}) + \sin\frac{5\pi}{2}\,\delta(t-\tfrac{1}{2})$$

(d) $$\int_{-\infty}^{t} e^{-2\tau-1}\,\delta(\tau-3)\,d\tau = \begin{cases} 0 & \text{if } t<3 \\ e^{-2\cdot 3-1} = e^{-7} & \text{if } t>3 \end{cases}$$

↑ peaks at $\tau=3$