PROBLEM:

A continuous-time system is defined by the input/output relation

$$y(t) = \int_{-\infty}^{t-2} x(\tau)d\tau$$

(a) Determine the impulse response, $h(t)$, of this system.

(b) Is this a stable system? Explain with a proof or counter-example.

(c) Is it a causal system? Explain with a proof or counter-example.

(d) Use the convolution integral to determine the output of the system when the input is $u(t)$, the unit step signal:

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Plot the output signal $y(t)$ versus t.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $$h(t) = \int_{-\infty}^{t-2} \delta(\tau)\, d\tau = \begin{cases} 0 & \text{if } t-2<0, \text{ i.e. } t<2 \\ 1 & \text{if } t-2>0, \text{ or } t>2 \end{cases}$$

$\uparrow$ peaks at $\tau=0$

$$\Rightarrow h(t) = u(t-2)$$

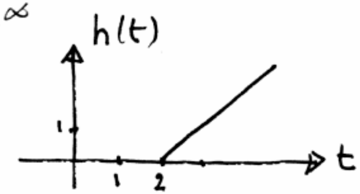
(b) $$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} u(t-2)\, dt = \int_{2}^{\infty} dt \quad \text{diverges!}$$

UNSTABLE

(c) CAUSAL since $h(t) = 0$ for $t<0$

(d) $$y(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau = \int_{-\infty}^{\infty} u(\tau-2)\, u(t-\tau)\, d\tau$$

$$= \int_{2}^{\min(2,t)} d\tau = \begin{cases} 0, & t<2 \\ t-2, & t>0 \end{cases}$$



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