PROBLEM:

A linear time-invariant system has impulse response:

$$h(t) = \begin{cases} e^{-t} & -1 \le t < 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $h(t - \tau)$ as a functions of τ for $t = -10$, 0, and 10.

(b) Find the output $y(t)$ when the input is $x(t) = \delta(t + 10)$.

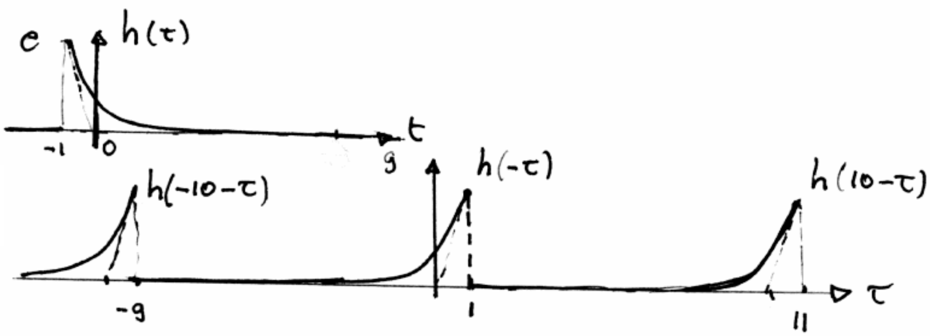
(c) Use the convolution integral to determine the output $y(t)$ when the input is

$$x(t) = \begin{cases} 1 & 0 \le t < 20 \\ 0 & \text{otherwise} \end{cases}$$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

(a)



(b) $$y(t) = \int_{-\infty}^{\infty} h(\tau)\,\delta(t-\tau+10)\,d\tau = \begin{cases} 0 & \text{if } t+10 \notin [-1,9] \\ \exp[-(t+10)] & \text{if } t+10 \in [-1,9] \end{cases}$$

peaks for $\tau = t+10$

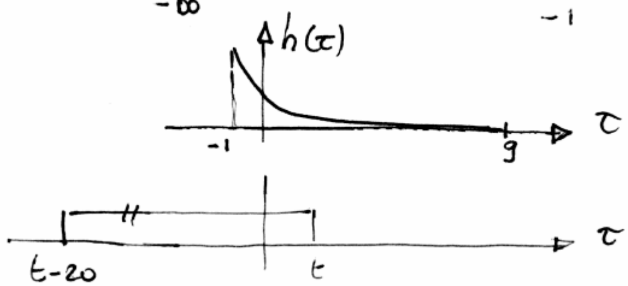
$$\therefore\ y(t) = \begin{cases} \exp[-(t+10)] & \text{if } t \in (-11,-1) \\ 0 & \text{else} \end{cases}$$

Also note that $\delta(t+10)$ is an impulse at $t=-10$, hence $y(t)$ is the corresponding shift of the impulse response.

$$y(t) = h(t+10)$$

(c) For $x(t) = \begin{cases} 1 & 0 \le t < 20 \\ 0 & \text{else} \end{cases}$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau = \int_{-1}^{9} e^{-\tau}\,x(t-\tau)\,d\tau$$



(c)

Use:
$$\int_0^x e^{-t}\,dt = 1 - e^{-x}$$

5 Regions

1. for $t < -1$, $y(t) = 0$

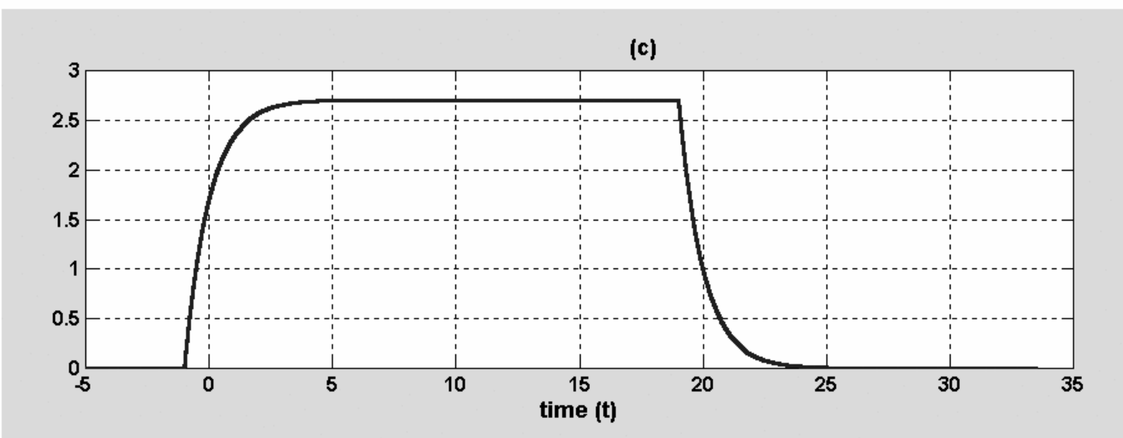
2. for $-1 \le t < 9$ $\quad y(t) = \int_{-1}^{t} e^{-\tau}\,d\tau = \left.\frac{e^{-\tau}}{-1}\right|_{-1}^{t} = e - e^{-t}$

3. for $t \ge 9$ and $t - 20 < -1$
   $9 \le t < 19$ $\quad y(t) = \int_{-1}^{9} e^{-\tau}\,d\tau = e - e^{-9}$

4. for $t - 20 \ge -1$ and $t - 20 < 9$
   $19 \le t < 29$ $\quad y(t) = \int_{t-20}^{9} e^{-\tau}\,d\tau = e^{-(t-20)} - e^{-9}$

5. for $t - 20 \ge 9 \Rightarrow t \ge 29$ $\quad y(t) = 0$

The signal $y(t)$ is given in the plot below:



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