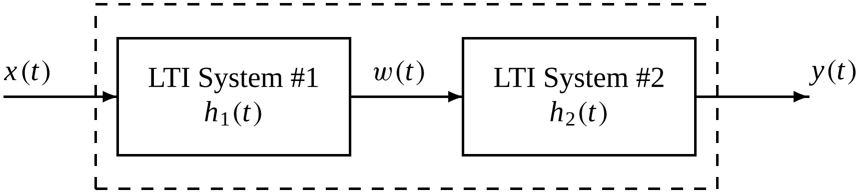
PROBLEM:



In the cascade of two LTI systems shown in the figure above, the first system has an impulse response

$$h_1(t) = u(t+3)$$

and the second system is described by the input/output relation

$$y(t) = \frac{dw(t)}{dt} - \pi w(t)$$

(a) Find the impulse response of the overall system; i.e., find the output $y(t) = h(t)$ when the input is $x(t) = \delta(t)$.

(b) Give a general expression for $y(t)$ in terms of $x(t)$ that holds for any input signal. Express your answer in terms of integrals, derivatives and delays. For example, you should try to get a form like:

$$y(t) = Ax(t-t_1) + B\frac{d}{dt}x(t-t_2) + C\int_{-\infty}^{t-t_3} x(\tau)d\tau$$

where the parameters A, B, C, t_1, t_2 and t_3 have specific numeric values.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

(a) If $x(t) = \delta(t) \Rightarrow w(t) = u(t+3)$

$$\Rightarrow y(t) = \frac{d}{dt}u(t+3) - \pi u(t+3)$$

$$= \delta(t+3) - \pi u(t+3)$$

$\therefore$ Impulse response of overall system $= \delta(t+3) - \pi u(t+3)$

(b) From (a): $y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$

$$= \int_{-\infty}^{\infty} \delta(\tau+3)\, x(t-\tau)\, d\tau - \pi \int_{-\infty}^{\infty} u(\tau+3)\, x(t-\tau)\, d\tau$$

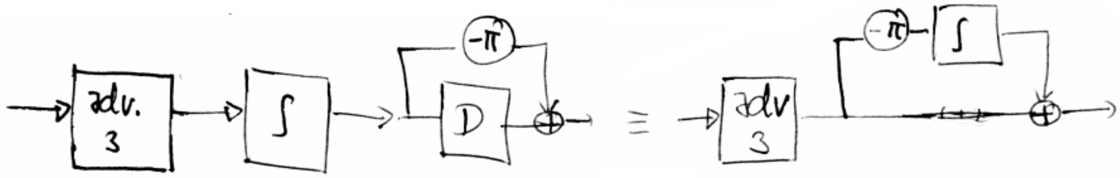
$$= x(t+3) - \pi \int_{-3}^{\infty} x(t-\tau)\, d\tau$$

letting $t-\tau = \tau'$ gives for the integral part:

$$\int_{\tau'=t+3}^{-\infty} x(\tau')\, d(-\tau') = \int_{-\infty}^{t+3} x(\tau')\, d\tau'$$

So: $$y(t) = x(t+3) - \pi \int_{-\infty}^{t+3} x(\tau)\, d\tau \qquad (*)$$

Remark: System 1 is an advance of 3 units, concatenated with an integrator. System 2 is a differentiator in paralel with a gain $(-\pi)$



from which (*) can easily be derived.