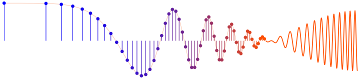
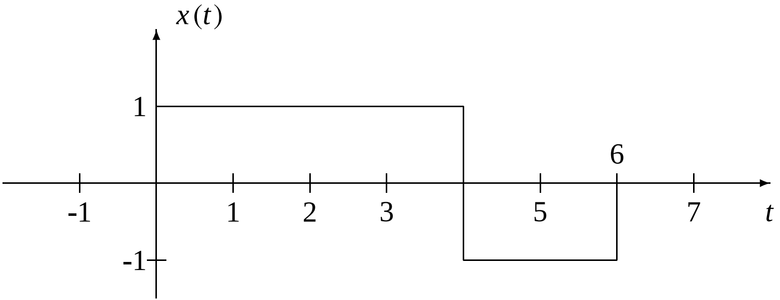
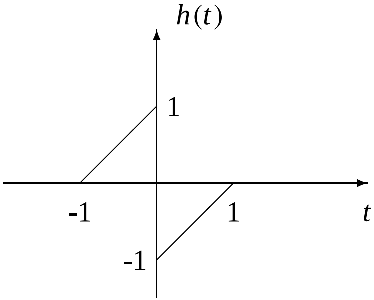


PROBLEM:

If the input $x(t)$ and the impulse response $h(t)$ of an LTI system are the following:





(a) Determine $y(0)$, the value of the output at $t = 0$.

(b) Find all the values of t for which the output $y(t) = 0$. There will be regions where $y(t) = 0$, but there might also be isolated points where $y(t) = 0$. *Note: You do not need to find $y(t)$ at any other values of t.*

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

Note that $x(t) = u(t) - 2u(t-4) + u(t-6)$

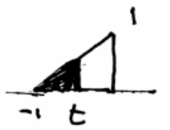
So, let's solve first $(h * u)(t) = g(t)$

$$\int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau = \int_{-\infty}^{t} h(\tau)\, d\tau$$

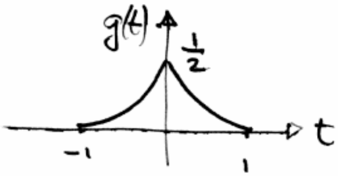
the integral of $h(\cdot)$!

$$\Rightarrow \quad g(t) = \begin{cases} 0, & t < -1 \\ \frac{(t+1)^2}{2}, & -1 < t < 0 \\ \frac{1}{2} - \left[\frac{1}{2} - \frac{(1-t)^2}{2}\right], & 0 < t < 1 \\ 0 & t > 1 \end{cases}$$

$\rightarrow$ area of little triangle



$\frac{(t-1)^2}{2} =$



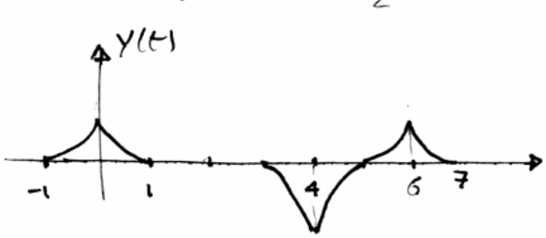
Thus: from time-invariance and linearity,

$$y(t) = g(t) - 2g(t-4) + g(t-6)$$

(a) In particular: for $t=0$

$$y(0) = g(0) - 2g(-4) + g(-6) = \frac{1}{2}$$

(b) $y(t) = 0$ for $\begin{cases} t < -1 \\ 1 < t < 3 \\ t = 5 \\ t > 7 \end{cases}$



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