PROBLEM:

Determine the z-transforms of the following:

(a) $x[n] = (\frac{1}{2})^n u[n]$.

(b) $x[n] = 3(\frac{1}{2})^n u[n] - (\frac{1}{3})^n u[n-1]$.

(c) $x[n] = \delta[n] + u[n]$.

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SOLUTION

(a)

$$X(z) = \sum_{n=-\infty}^{+\infty} \left(\frac{1}{2}\right)^n u[n]\, z^{-n}$$

$$= \sum_{n=0}^{+\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$$= \frac{1}{1 - \frac{1}{2} z^{-1}}$$

(b)

$$X(z) = \sum_{n=-\infty}^{+\infty} \left(3\left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{3}\right)^n u[n-1]\right) z^{-n}$$

$$= \sum_{n=0}^{+\infty} 3 \cdot \left(\frac{1}{2}\right)^n z^{-n} - \sum_{n=1}^{+\infty} \left(\frac{1}{3}\right)^n z^{-n}$$

$$= \frac{3}{1 - \frac{1}{2} z^{-1}} - \frac{\frac{1}{3} z^{-1}}{1 - \frac{1}{3} z^{-1}}$$

$$= \frac{3 - \frac{4}{3} z^{-1} + \frac{1}{6} z^{-2}}{(1 - \frac{1}{2} z^{-1})(1 - \frac{1}{3} z^{-1})}$$

(c)

$$X(z) = \sum_{n=-\infty}^{+\infty} \left(\delta[n] + u[n]\right) z^{-n}$$

$$= \sum_{n=0} \delta[n] z^{-n} + \sum_{n=0}^{+\infty} u[n] z^{-n}$$

$$= 1 + \frac{1}{1 - z^{-1}}$$

$$= \frac{2 - z^{-1}}{1 - z^{-1}}$$

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