PROBLEM:

Determine the inverse z-transforms of the following:

(a) $H_a(z) = \dfrac{1 + 2z^{-2}}{1 - 0.25z^{-1}}$.

(b) $H_b(z) = \dfrac{3}{1 - 0.3z^{-1}} - \dfrac{2}{1 + 0.7z^{-1}}$.

(c) $H_c(z) = \dfrac{1 + 2z^{-1}}{1 - 0.4z^{-1} - 0.32z^{-2}}$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a)
$$H_a(z) = \frac{1+2z^{-2}}{1-0.25z^{-1}} = \frac{1}{1-0.25z^{-1}} + \frac{2z^{-2}}{1-0.25z^{-1}}$$

$$h_a[n] = (0.25)^n u[n] + 2\,(0.25)^{n-2}\,u[n-2]$$

(b)
$$H_b(z) = \frac{3}{1-0.3z^{-1}} - \frac{2}{1+0.7z^{-1}}$$

$$h_b[n] = 3\cdot(0.3)^n u[n] - 2\,(-0.7)^n u[n]$$

(c)
$$H_c(z) = \frac{1+2z^{-1}}{1-0.4z^{-1}-0.32z^{-2}} = \frac{1+2z^{-1}}{(1-0.8z^{-1})(1+0.4z^{-1})}$$

$$= \frac{7/3}{1-0.8z^{-1}} + \frac{-4/3}{1+0.4z^{-1}}$$

$$h_c[n] = 7/3\cdot(0.8)^n u[n] - 4/3\cdot(-0.4)^n u[n]$$

To decompose $\frac{1+2z^{-1}}{(1-0.8z^{-1})(1+0.4z^{-1})}$ into $\frac{7/3}{1-0.8z^{-1}} + \frac{-4/3}{1+0.4z^{-1}}$,
set
$$\frac{1+2z^{-1}}{(1-0.8z^{-1})(1+0.4z^{-1})} = \frac{a}{1-0.8z^{-1}} + \frac{b}{1+0.4z^{-1}} \quad (*)$$

Then the right-side of (*) is
$$\frac{a+b+(0.4a-0.8b)z^{-1}}{(1-0.8z^{-1})(1+0.4z^{-1})},$$

which should be equal to the left-side of (*), that is
$$a+b=1 \;\&\; 0.4a-0.8b=2 \implies a=7/3 \;\&\; b=-4/3$$

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