PROBLEM:

For each of the difference equations below, determine the poles and zeros of the corresponding system function, $H(z)$. Plot the poles **(X)** and zeros **(O)** in the complex z-plane.

$$\mathcal{S}_1: \quad y[n] = 0.45y[n-1] + x[n] - x[n-1]$$

$$\mathcal{S}_2: \quad y[n] = -0.7y[n-1] + x[n] + x[n-1]$$

$$\mathcal{S}_3: \quad y[n] = -0.25y[n-1] + 0.75y[n-2] + x[n]$$

$$\mathcal{S}_4: \quad y[n] = x[n] + \tfrac{1}{4}x[n-1] - \tfrac{3}{4}x[n-2]$$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

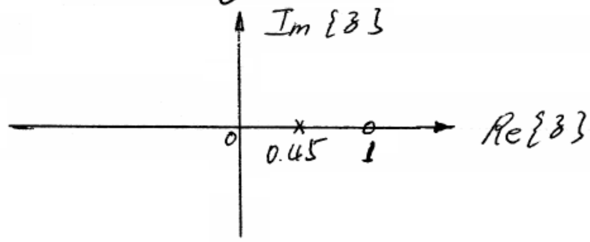
S_1 : Take z-transform to the both sides of the equation.

$$Y(z) = 0.45 z^{-1} Y(z) + X(z) - z^{-1} X(z)$$

Then $$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{1 - 0.45 z^{-1}}$$

pole: 0.45

zero: 1



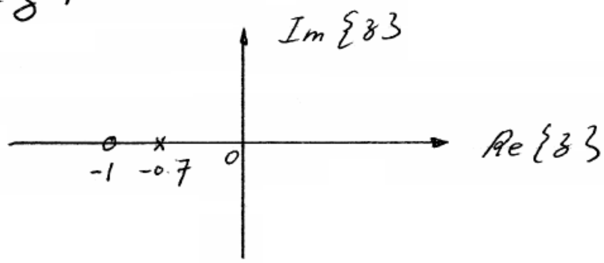
S_2 :

$$Y(z) = -0.7 z^{-1} Y(z) + X(z) + z^{-1} X(z)$$

$$H(z) = \frac{1 + z^{-1}}{1 + 0.7 z^{-1}}$$

pole: −0.7

zero: −1



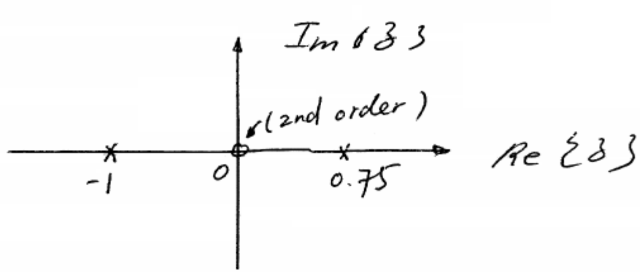
S_3 :

$$Y(z) = -0.25 z^{-1} Y(z) + 0.75 z^{-2} Y(z) + X(z)$$

$$H(z) = \frac{1}{1 + 0.25 z^{-1} - 0.75 z^{-2}} = \frac{z^2}{(z+1)(z-0.75)}$$

poles: −1, 0.75

zeros: 0 (2nd order)



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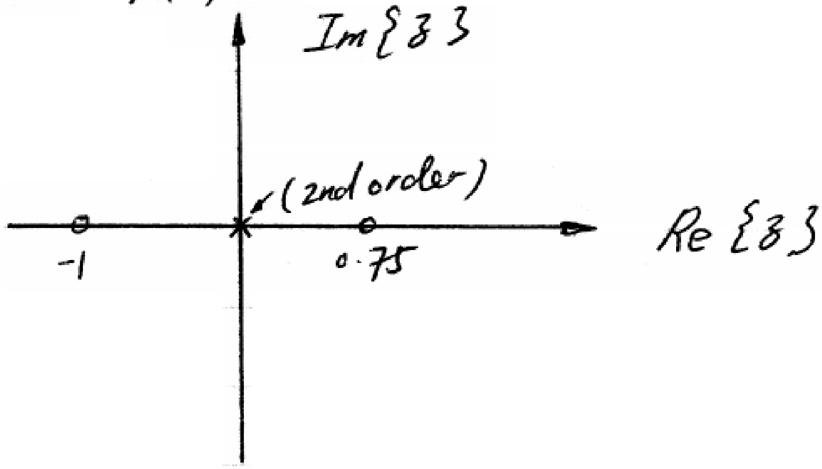
S_4 :

$$Y(z) = X(z) + \frac{1}{4} z^{-1} X(z) - \frac{3}{4} z^{-2} X(z)$$

$$H(z) = 1 + \frac{1}{4} z^{-1} - \frac{3}{4} z^{-2} = \frac{(z - 3/4)(z+1)}{z^2}$$

poles : 0 (2nd order)

zeros : $3/4$, -1



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