PROBLEM:

An LTI system has the following system function:

$$H(z) = \frac{1 + z^{-2}}{1 + 0.3z^{-1}}.$$

The following questions cover most of the ways available for analyzing IIR discrete-time systems.

(a) Plot the poles and zeros of $H(z)$ in the z-plane.

(b) Use z-transforms to determine the impulse response $h[n]$ of the system; i.e., the output of the system when the input is $x[n] = \delta[n]$.

(c) Determine an expression for the frequency response $H(e^{j\hat{\omega}})$ of the system.

(d) Use the frequency response function to determine the output $y_1[n]$ of the system when the input is

$$x_1[n] = 2\cos(\pi n) \qquad -\infty < n < \infty.$$

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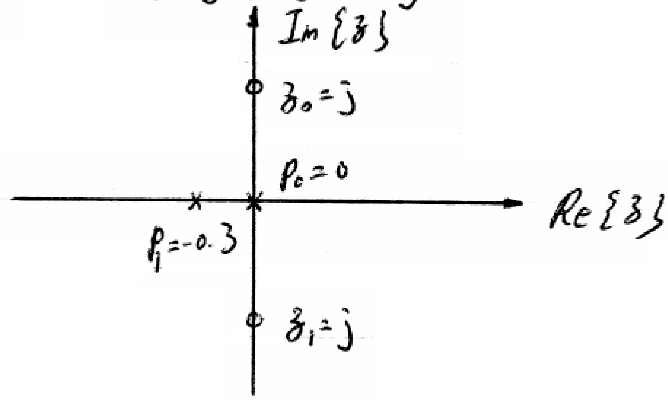
SOLUTION

(a)

$$H(z) = \frac{1+z^{-2}}{1+0.3z^{-1}} = \frac{(z+j)(z-j)}{z(z+0.3)}$$

Poles: $p_0 = 0,\ p_1 = -0.3.$

zeros: $z_0 = j,\ z_1 = -j$



(b)

$$h[n] = Z^{-1}\{H(z)\}$$

$$= Z^{-1}\left\{\frac{1+z^{-2}}{1+0.3z^{-1}}\right\}$$

$$= Z^{-1}\left\{\frac{1}{1+0.3z^{-1}}\right\} + Z^{-1}\left\{\frac{z^{-2}}{1+0.3z^{-1}}\right\}$$

$$= (-0.3)^{n}u[n] + (-0.3)^{n-2}u[n-2]$$

(c)

$$H(e^{j\hat{\omega}}) = \frac{1+e^{-j2\hat{\omega}}}{1+0.3e^{-j\hat{\omega}}}$$

(d) $$|H(e^{j\pi})| = \left|\frac{1+e^{-j2\pi}}{1+0.3e^{-j\pi}}\right| = \left|\frac{1+1}{1-0.3}\right| = 20/7$$

$$\angle H(e^{j\pi}) = \angle\frac{1+1}{1-0.3} = 0$$

$$y_1[n] = 2\cdot|H(e^{j\pi})|\cos(\pi n + \angle H(e^{j\pi}))$$

$$= 2\times 20/7\cdot\cos(\pi n + 0)$$

$$= 40/7\cos(\pi n)$$

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