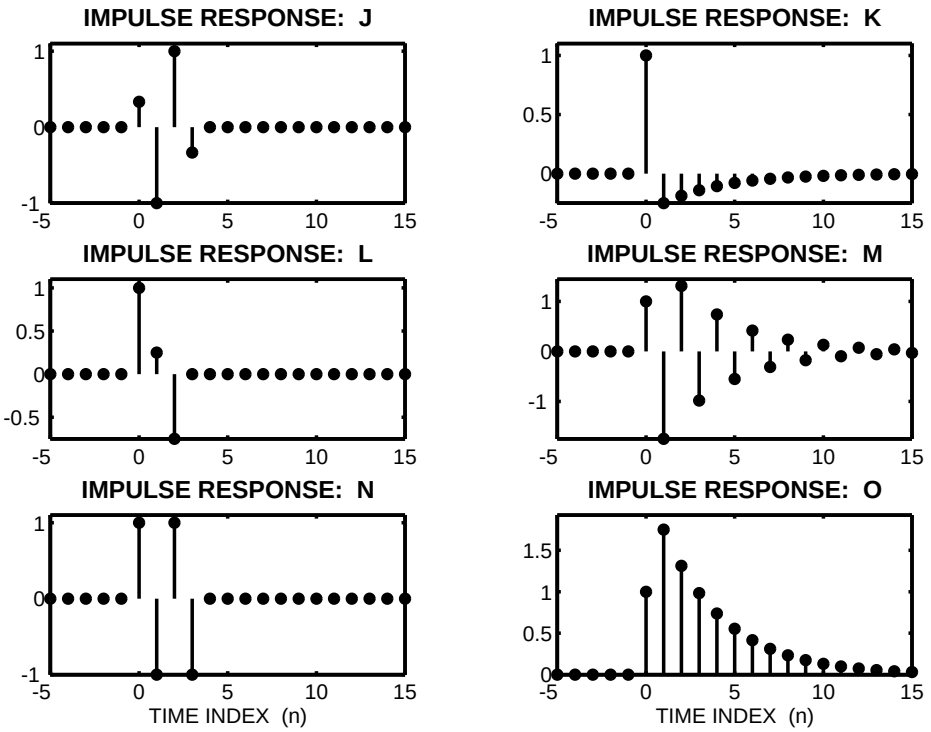
PROBLEM:



For each of the impulse-response plots (J, K, L, M, N, O), determine which one of the following systems[1] (specified by either an $H(z)$ or a difference equation) matches the impulse response. In addition, derive a formula for the impulse response, $h[n]$, for $\mathcal{S}_1$ and $\mathcal{S}_4$.

$\mathcal{S}_1$: $y[n] = 0.4y[n-1] + x[n] + x[n-1]$

$\mathcal{S}_2$: $H(z) = \dfrac{1+z^{-1}}{1-0.75z^{-1}}$

$\mathcal{S}_3$: $y[n] = -0.75y[n-1] + x[n] - x[n-1]$

$\mathcal{S}_4$: $H(z) = \dfrac{1-z^{-1}}{1-0.75z^{-1}}$

$\mathcal{S}_5$: $y[n] = x[n] - x[n-1] + x[n-2]$

$\mathcal{S}_6$: $H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

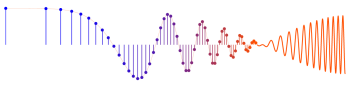
$\mathcal{S}_7$: $y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

$\mathcal{S}_8$: $H(z) = \frac{1}{3}(1-z^{-1})^3$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

[1] These 8 systems are exactly the same as the next matching problems.



Impulse J: FIR with non-zero values at $n=0, 1, 2, 3$

S_8, $H(z) = \frac{1}{3}(1-z^{-1})^3 = \frac{1}{3} - z^{-1} + z^{-2} - \frac{1}{3}z^{-3}$

$h[n] = \frac{1}{3}\delta[n] - \delta[n-1] + \delta[n-2] - \frac{1}{3}\delta[n-3]$

Impulse K: IIR with a positive decaying factor since $n=1$

S_4, $H(z) = \frac{1-z^{-1}}{1-0.75z^{-1}} = 1 - \frac{0.25z^{-1}}{1-0.75z^{-1}}$

$h[n] = \delta[n] - 0.25 \cdot (0.75)^{n-1} u[n-1]$

Impulse L: FIR with three non-zero values at $n = 0, 1, 2$, two positive & one negativ

S_7, $y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

$h[n] = \delta[n] + \frac{1}{4}\delta[n-1] - \frac{3}{4}\delta[n-2]$

Impulse M: IIR with a NEGATIVE decaying factor (since $h[n]$ oscillates).

S_3, $y[n] = -0.75\,y[n-1] + x[n] - x[n-1]$

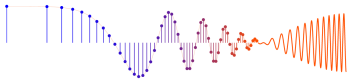
$H(z) = \frac{1-z^{-1}}{1+0.75z^{-1}} = 1 - \frac{1.75z^{-1}}{1+0.75z^{-1}}$

$h[n] = \delta[n] - 1.75(-0.75)^{n-1} u[n-1]$

Impulse N: FIR with four non-zero values at $n = 0, 1, 2, 3$

S_6, $H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

$h[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$



Impulse 0 : IIR with a positive decaying factor since $n = 1$

S_2 : $H(z) = \frac{1+z^{-1}}{1-0.75z^{-1}} = 1 + \frac{1.75z^{-1}}{1-0.75z^{-1}}$

$$h[n] = \delta[n] + 1.75\,(0.75)^{n-1}u[n-1]$$

Impulse response of S_1 :

$$H(z) = \frac{1+z^{-1}}{1-0.4z^{-1}} = 1 + \frac{1.4z^{-1}}{1-0.4z^{-1}}$$

$$h[n] = \delta[n] + 1.4 \cdot (0.4)^{n-1}u[n-1]$$

Impulse response of S_4 :

$$h[n] = \delta[n] - 0.25 \cdot (0.75)^{n-1}u[n-1]$$