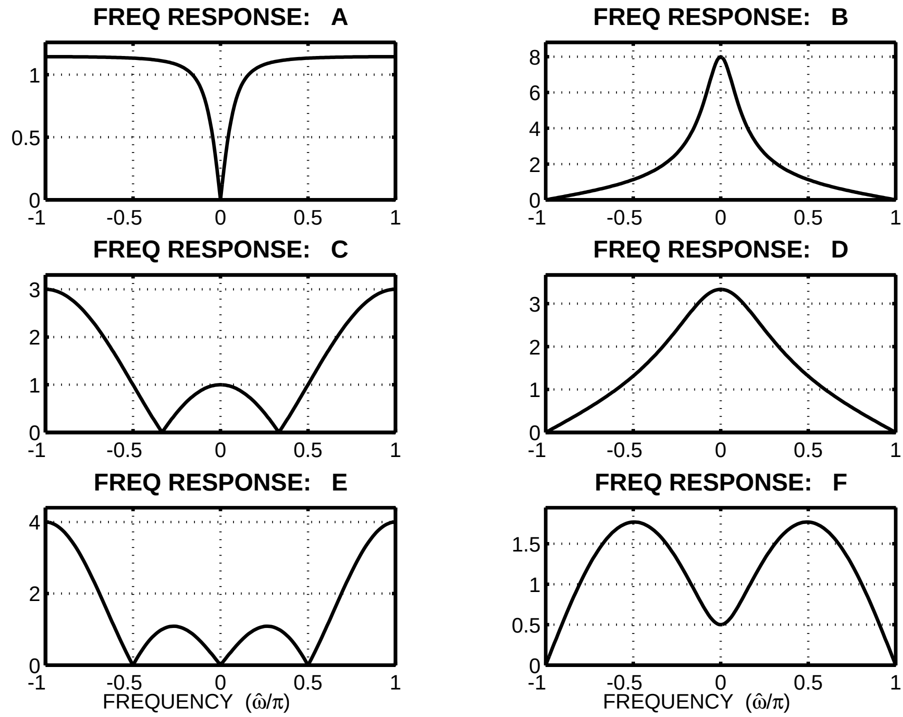
PROBLEM:



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems[1] (specified by either an $H(z)$ or a difference equation) matches the frequency response (magnitude only). NOTE: frequency axis is **normalized**; it is $\hat{\omega}/\pi$. In addition, derive a formula for the magnitude-squared of the frequency response, $|H(e^{j\hat{\omega}})|^2$, for $\mathcal{S}_3$ and $\mathcal{S}_4$.

$\mathcal{S}_1$: $\quad y[n] = 0.4y[n-1] + x[n] + x[n-1]$

$\mathcal{S}_2$: $\quad H(z) = \dfrac{1+z^{-1}}{1-0.75z^{-1}}$

$\mathcal{S}_3$: $\quad y[n] = -0.75y[n-1] + x[n] - x[n-1]$

$\mathcal{S}_4$: $\quad H(z) = \dfrac{1-z^{-1}}{1-0.75z^{-1}}$

$\mathcal{S}_5$: $\quad y[n] = x[n] - x[n-1] + x[n-2]$

$\mathcal{S}_6$: $\quad H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

$\mathcal{S}_7$: $\quad y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

$\mathcal{S}_8$: $\quad H(z) = \frac{1}{3}(1-z^{-1})^3$

McClellan, Schafer and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.

SOLUTION

[1] These 8 systems are exactly the same as the previous matching problems.

Response A: $H(e^{j0}) = 0$ & $|H(e^{j\pm\pi})| = 1.1^{+}$

S_4 : $H(e^{j0}) = 0$, $H(e^{j\pm\pi}) = 8/7$

Response B: $H(e^{j\pm\pi}) = 0$ & $|H(e^{j0})| = 8$ (Maximum)

S_2 : $H(e^{j\pm\pi}) = 0$, $H(e^{j0}) = 8$

Response C: $H(e^{j\pm .3\pi}) = 0$, $H(e^{j\pm\pi}) = 3$ (Maximum)

S_5 : $H(z) = 1 - z^{-1} + z^{-2}$ $H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$

$H(e^{j\pm\pi}) = 3$, $H(e^{j\pm\pi/3}) = 0$

Response D: $H(e^{j\pm\pi}) = 0$, $H(e^{j0}) = 3.5$ (Maximum)

S_1 : $H(z) = \dfrac{1+z^{-1}}{1-0.4z^{-1}}$, $H(e^{j\hat{\omega}}) = \dfrac{1+e^{-j\hat{\omega}}}{1-0.4e^{-j\hat{\omega}}}$

$H(e^{-j0}) = 20/6$ (≈ 3.3), $H(e^{j\pm\pi}) = 0$

Response E: $H(e^{j\hat{\omega}}) = 0$ at $\hat{\omega} = \pm 0.5\pi, 0$

$H(e^{j\pi}) = 4$ (max)

S_6 : $H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$

$= (1-e^{-j\hat{\omega}})(1-e^{j\pi/2}e^{-j\hat{\omega}})(1-e^{-j\pi/2}e^{-j\hat{\omega}})$

Response F: $H(e^{j\pm\pi}) = 0$, $H(e^{j0}) = 0.5$

S_7 : $H(e^{j\hat{\omega}}) = 1 + \frac{1}{4}e^{-j\hat{\omega}} - \frac{3}{4}e^{-j2\hat{\omega}}$

$= (1+e^{-j\hat{\omega}})(1-\frac{3}{4}e^{-j\hat{\omega}})$

S_3 : $$H(e^{j\hat{\omega}}) = \frac{1-e^{-j\hat{\omega}}}{1+0.75e^{-j\hat{\omega}}}$$

$$|H(e^{j\hat{\omega}})|^2 = \left| \frac{1-(\cos\hat{\omega} - j\sin\hat{\omega})}{1+0.75(\cos\hat{\omega} - j\sin\hat{\omega})} \right|^2$$

$$= \frac{(1-\cos\hat{\omega})^2 + \sin^2\hat{\omega}}{(1+0.75\cos\hat{\omega})^2 + (0.75)^2\sin^2\hat{\omega}}$$

$$= \frac{1-2\cos\hat{\omega} + \cos^2\hat{\omega} + \sin^2\hat{\omega}}{1+1.5\cos\hat{\omega} + 0.5625\cos^2\hat{\omega} + 0.5625\sin^2\hat{\omega}}$$

$$= \frac{2(1-\cos\hat{\omega})}{1.5625 + 1.5\cos\hat{\omega}}$$

S_4 : $$H(e^{j\hat{\omega}}) = \frac{1-e^{-j\hat{\omega}}}{1-0.75e^{-j\hat{\omega}}}$$

$$|H(e^{j\hat{\omega}})|^2 = \left| \frac{1-(\cos\hat{\omega} - j\sin\hat{\omega})}{1-0.75(\cos\hat{\omega} - j\sin\hat{\omega})} \right|^2$$

$$= \frac{(1-\cos\hat{\omega})^2 + \sin^2\hat{\omega}}{(1-0.75\cos\hat{\omega})^2 + 0.75^2\sin^2\hat{\omega}}$$

$$= \frac{2(1-\cos\hat{\omega})}{1.5625 - 1.5\cos\hat{\omega}}$$