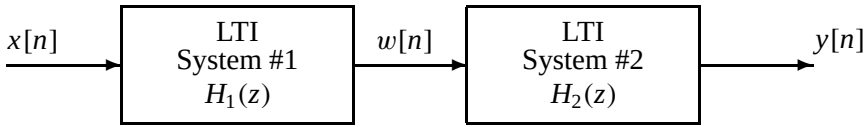
## PROBLEM:

Consider the following cascade system:




where

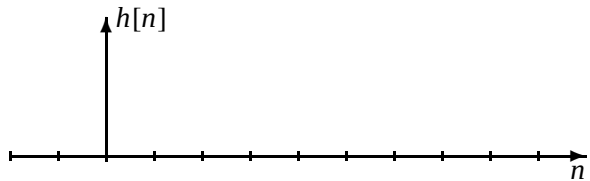
$$H_1(z) = 2 - z^{-1} - z^{-2} \quad \text{and} \quad H_2(z) = 1 + \tfrac{1}{2}z^{-1}$$

(a) If the input $x[n]$ is a step,

$$x[n] = \begin{cases} 1 & \text{for } 0 \le n \\ 0 & \text{for } n < 0 \end{cases}$$

Find the output of the **first filter**, $w[n]$.

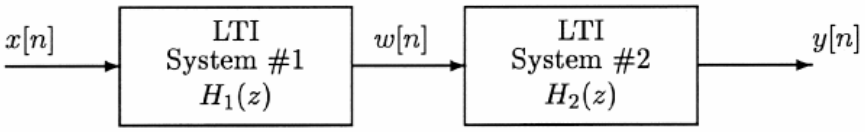
(b) Find and plot the impulse response $h[n]$ of the overall system.




McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458. 

SOLUTION

Consider the following cascade system:




where

$$H_1(z) = 2 - z^{-1} - z^{-2} \quad \text{and} \quad H_2(z) = 1 + \tfrac{1}{2}z^{-1}$$

(a) If the input $x[n]$ is a step,

$$x[n] = \begin{cases} 1 & ; \quad 0 \le n \\ 0 & ; \quad n < 0 \end{cases}$$

Find the output of the **first filter**, $w[n]$.

SINCE $h_1[n] = 2\delta[n] - \delta[n-1] - \delta[n-2]$, then

$$w[n] = h_1[n] * x[n] = \{2\delta[n] - \delta[n-1] - \delta[n-2]\} * u[n]$$

$$= 2u[n] - u[n-1] - u[n-2]$$

$$= \begin{cases} 2 & n=0 \\ 1 & n=1 \\ 0 & \text{else} \end{cases}$$

(b) Find and plot the impulse response $h[n]$ of the overall system.

$$H(z) = H_1(z)H_2(z) = (2 - z^{-1} - z^{-2})(1 + \tfrac{1}{2}z^{-1})$$

$$= 2 + (1-1)z^{-1} + (-1-\tfrac{1}{2})z^{-2} - \tfrac{1}{2}z^{-3}$$

$$= 2 - \tfrac{3}{2}z^{-2} - \tfrac{1}{2}z^{-3}$$

SO

$$h[n] = 2\delta[n] - \tfrac{3}{2}\delta[n-2] - \tfrac{1}{2}\delta[n-3]$$