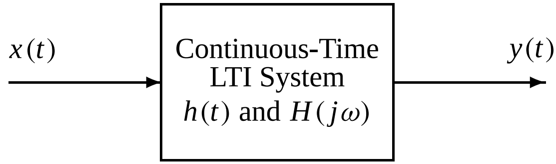
PROBLEM:

The input to the LTI system shown below is a periodic signal $x(t)$ that has a period $\boxed{T_0 = 10}$ seconds.



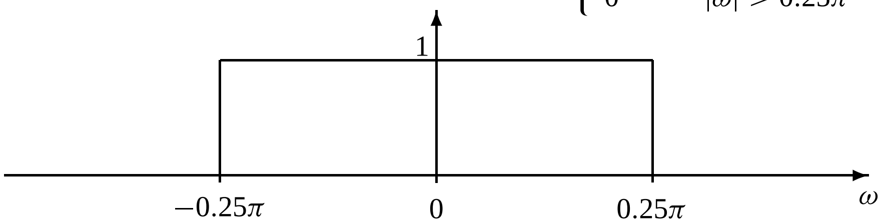
The Fourier series representation for the input $x(t)$ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad \text{where} \qquad a_k = \begin{cases} 0.5 & k = 0 \\ \dfrac{3\sin(\pi k/2)}{\pi k} & k \neq 0. \end{cases}$$

(a) What is the fundamental frequency ω_0 of the input signal $x(t)$? $\boxed{\omega_0 = \rule{4cm}{0.4pt} \text{ rad/sec}}$

(b) Suppose that the frequency response of the system is an ideal lowpass filter as illustrated below.

$$H(j\omega) = \begin{cases} 1 & |\omega| < 0.25\pi \\ 0 & |\omega| > 0.25\pi \end{cases}$$

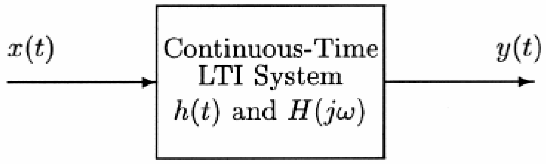


Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. (Your answer should be expressed in terms of only real quantities). Justify your answer by sketching the Fourier transform of $y(t)$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

The input to the LTI system shown below is a periodic signal $x(t)$ that has a period $\boxed{T_0 = 1/3}$ seconds.



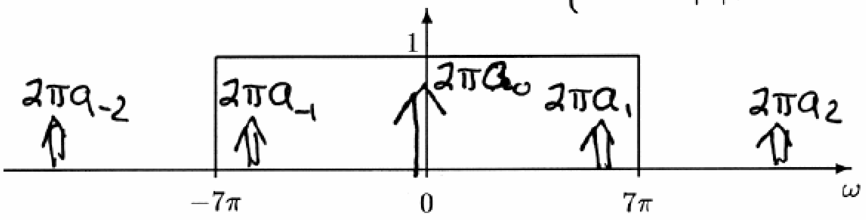
The Fourier series representation for the input $x(t)$ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad \text{where} \qquad a_k = \begin{cases} 3 & k = 0 \\ \dfrac{\sin(\pi k/2)}{4\pi k} & k \neq 0. \end{cases}$$

(a) What is the fundamental frequency ω_0 of the input signal $x(t)$? $\boxed{\omega_0 = \underline{2\pi/T_0 = 6\pi} \text{ rad/sec}}$

(b) Suppose that the frequency response of the system is an ideal lowpass filter as illustrated below.

$$H(j\omega) = \begin{cases} 1 & |\omega| < 7\pi \\ 0 & |\omega| > 7\pi \end{cases}$$



Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. (Your answer should be expressed in terms of only real quantities). Justify your answer by sketching the Fourier transform of $y(t)$.

The spectrum of $x(t)$ is plotted above. Note that the filter only passes three terms in the FS expansion,

$$y(t) = a_0 + a_1 e^{j\omega_0 t} + a_{-1} e^{-j\omega_0 t}$$

$$= a_0 + 2a_1 \cos(\omega_0 t)$$

$$= 3 + \frac{1}{2\pi}\cos(6\pi t)$$

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