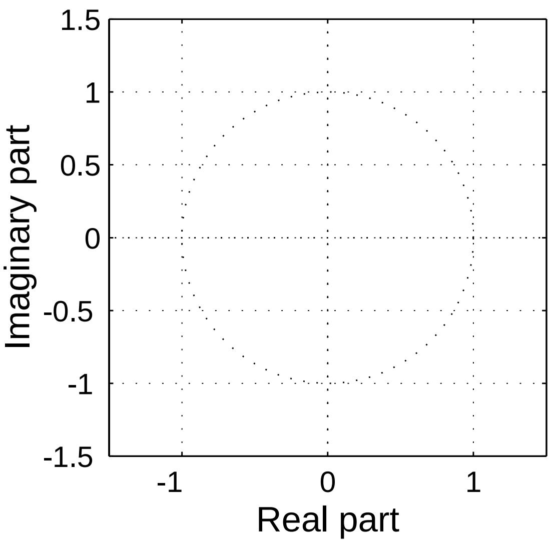
PROBLEM:

A discrete-time system (FIR filter) is defined by the following z-transform system function:

$$H(z) = (1 - 0.5z^{-1})(1 - e^{j3\pi/4}z^{-1})(1 - e^{-j3\pi/4}z^{-1})$$

(a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system. Give the numerical values of all filter coefficients.

(b) Determine *all* the zeros of $H(z)$ and plot them in the z-plane.



(c) If the input is of the form $x[n] = A\sin(\hat{\omega}_0 n + \phi)$, for what value of frequency $\hat{\omega}_0$ (in the range $0 < \hat{\omega}_0 < \pi$) will the filter completely remove the sinusoidal component? **EXPLAIN your answer.**

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

A discrete-time system (FIR filter) is defined by the following z-transform system function:

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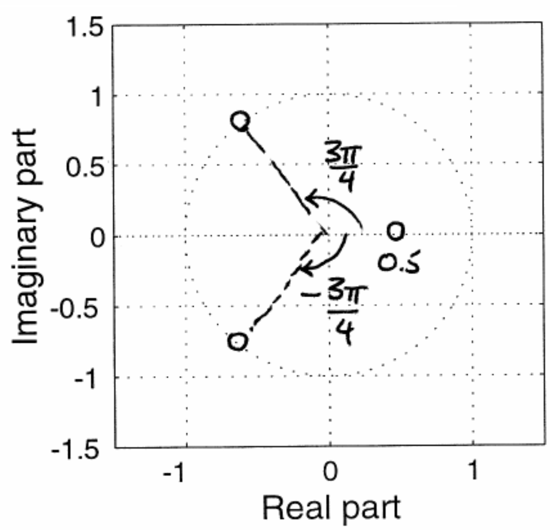
(a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system. Give the numerical values of all filter coefficients.

$$H(z) = (1 - 0.5z^{-1})(1 + 1.414z^{-1} + z^{-2})$$

$$= 1 + 0.914z^{-1} + 0.293z^{-2} - 0.5z^{-3}$$

$$\boxed{y[n] = x[n] + 0.914\,x[n-1] + 0.293\,x[n-2] - 0.5\,x[n-3]}$$

(b) Determine *all* the zeros of $H(z)$ and plot them in the z-plane.



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$$y[n] = A\,|H(e^{j\hat{\omega}_0})|\sin(\hat{\omega}_0 n + \phi + \angle H(e^{j\hat{\omega}_0}))$$

FOR $y[n]$ TO BE ZERO, $|H(e^{j\hat{\omega}_0})|$ SHOULD BE ZERO, WHICH REQUIRES A ZERO OF THE SYSTEM FN. ON THE UNIT CIRCLE AT THE APPROPRIATE FREQUENCY.

$$\boxed{\hat{\omega}_0 = \frac{3\pi}{4} \text{ RAD.}}$$

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