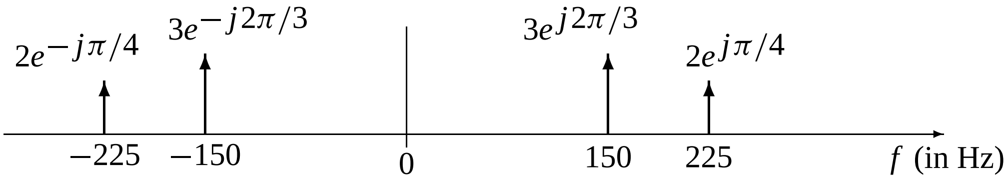
PROBLEM:

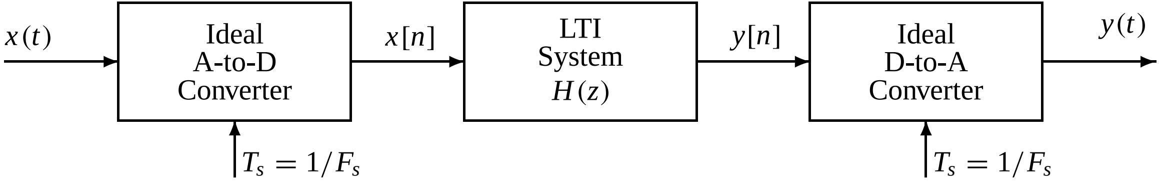
The input $x(t)$ to the A-to-D converter is specified by its spectrum in the figure below:



The z-transform for the LTI system is

$$H(z) = \left(1 - e^{j\pi/4} z^{-1}\right)\left(1 - e^{-j\pi/4} z^{-1}\right)$$

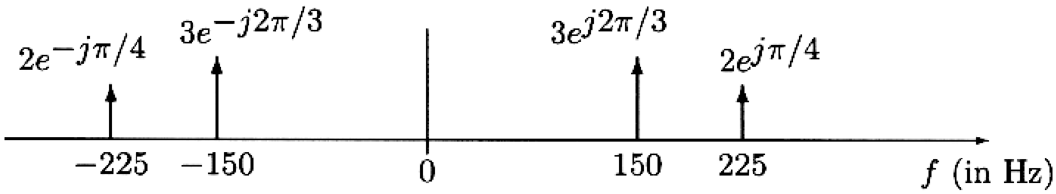
If $F_s = 200$ samples/second, determine an expression for $y(t)$, the output of the D-to-A converter.



McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

The input $x(t)$ to the A-to-D converter is specified by its spectrum in the figure below:



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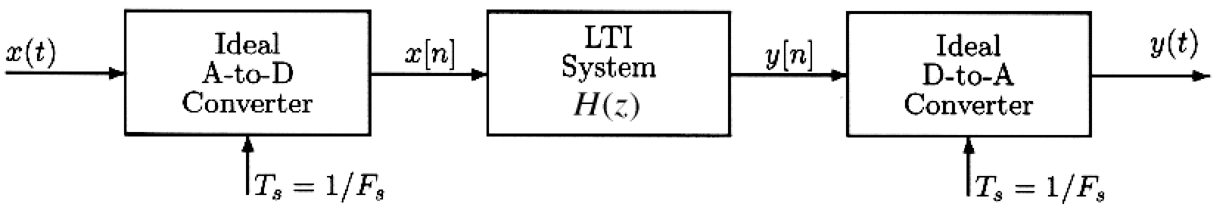

$$x(t) = 6\cos(2\pi(150)t + 2\pi/3) + 4\cos(2\pi(225)t + \pi/4)$$

Replace t with $n/F_s = n/200$ to do A/D:

$$x[n] = 6\cos\Big(\underbrace{2\pi\left(\tfrac{150}{200}\right)n}_{2\pi\left(\frac{3}{4}\right)n} + \tfrac{2\pi}{3}\Big) + 4\cos\Big(\underbrace{2\pi\left(\tfrac{225}{200}\right)n}_{2\pi\left(1\frac{1}{8}\right)n} + \pi/4\Big)$$

FOLDS to $2\pi(-\frac{1}{4})n$

same as $2\pi(\frac{1}{8})n$

$$x[n] = 6\cos\left(2\pi(-\tfrac{1}{4})n + \tfrac{2\pi}{3}\right) + 4\cos\left(2\pi(\tfrac{1}{8})n + \pi/4\right)$$

$$= \cos\left(2\pi(\tfrac{1}{4})n - 2\pi/3\right) + 4\cos\left(2\pi(\tfrac{1}{8})n + \pi/4\right)$$

Eval $H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/2$ & $\pi/4$.

$$H(\pi/2) = (1 - e^{j\pi/4}e^{-j\pi/2})(1 - e^{-j\pi/4}e^{-j\pi/2}) = \sqrt{2}\,e^{j\pi/2}$$

$$H(\pi/4) = (\underbrace{1 - e^{j\pi/4}e^{-j\pi/4}}_{\text{ZERO}})(1 - e^{-j\pi/4}e^{-j\pi/4}) = 0$$

$$\Rightarrow y[n] = 6\sqrt{2}\cos\left(2\pi(\tfrac{1}{4})n - 2\pi/3 + \tfrac{\pi}{2}\right)$$

REPLACE n with $F_s t$
$n \leftarrow 200t$

$$y(t) = 6\sqrt{2}\cos\left(2\pi(50)t - \pi/6\right)$$

McClellan, Schafer, and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.