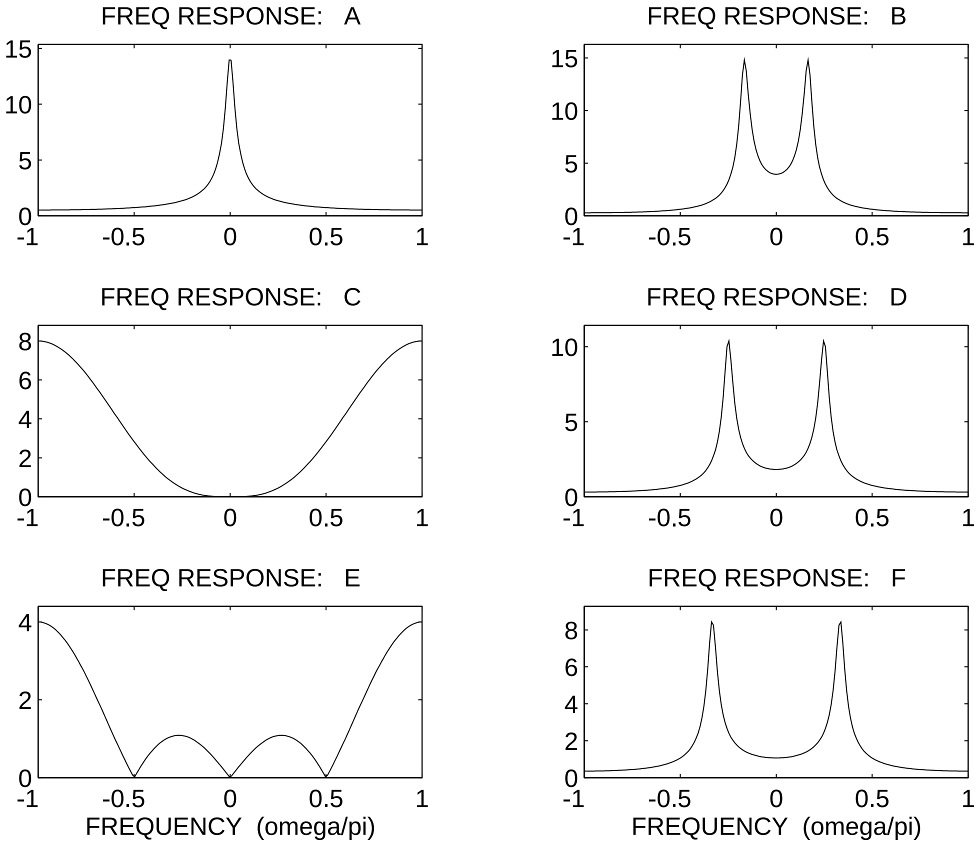
PROBLEM:



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $H(z)$ or a difference equation) matches the frequency response. NOTE: frequency axis is normalized; it is $\hat{\omega}/\pi$.

$$\mathcal{S}_0: \quad y[n] = 0.93y[n-1] + x[n-1]$$

$$\mathcal{S}_1: \quad H(z) = \frac{z^{-1}}{1 - 1.315z^{-1} + 0.8649z^{-2}}$$

$$\mathcal{S}_2: \quad H(z) = \frac{z^{-1}}{1 - 1.61z^{-1} + 0.8649z^{-2}}$$

$$\mathcal{S}_3: \quad H(z) = \frac{z^{-1}}{1 - 0.93z^{-1} + 0.8649z^{-2}}$$

$$\mathcal{S}_4: \quad H(z) = (1 - z^{-1})^3$$

$$\mathcal{S}_5: \quad H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$$

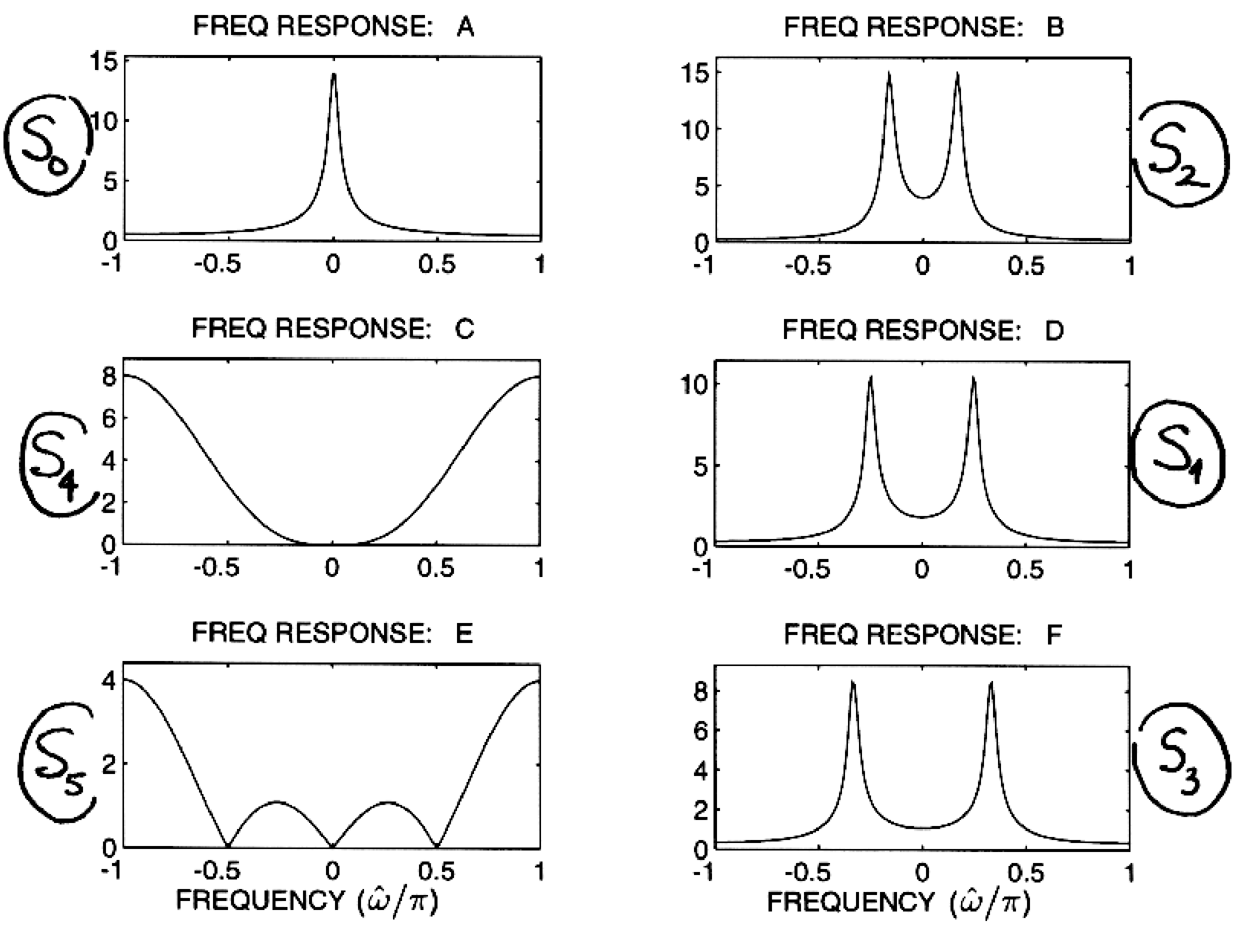
$$\mathcal{S}_6: \quad y[n] = x[n] - x[n-3]$$

$$\mathcal{S}_7: \quad y[n] = \sum_{k=0}^{3} x[n-k]$$

$$\mathcal{S}_8: \quad y[n] = x[n] + 2x[n-1] + 3x[n-2] + 2x[n-3] + x[n-4]$$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $H(z)$ or a difference equation) matches the frequency response. NOTE: frequency axis is normalized; it is $\hat{\omega}/\pi$.

$\mathcal{S}_0$: $y[n] = 0.93y[n-1] + x[n-1]$ $\Rightarrow$ $H(e^{j\hat{\omega}}) = \dfrac{e^{-j\hat{\omega}}}{1-0.93e^{-j\hat{\omega}}}$ $\Rightarrow$ peak at $\hat{\omega}=0$

$\mathcal{S}_1$: $H(z) = \dfrac{z^{-1}}{1-1.315z^{-1}+0.8649z^{-2}}$ $\Rightarrow$ pole $\angle$ is 45° $\Rightarrow$ peak at $\hat{\omega}=\pi/4$

$\mathcal{S}_2$: $H(z) = \dfrac{z^{-1}}{1-1.61z^{-1}+0.8649z^{-2}}$ $\Rightarrow$ pole $\angle$ is 30° $\Rightarrow$ peak at $\hat{\omega}=\pi/6$

$\mathcal{S}_3$: $H(z) = \dfrac{z^{-1}}{1-0.93z^{-1}+0.8649z^{-2}}$ $\Rightarrow$ pole $\angle$ is 60° $\Rightarrow$ peak at $\hat{\omega}=\pi/3$

$\mathcal{S}_4$: $H(z) = (1-z^{-1})^3$ $\Rightarrow$ $H(e^{j\hat{\omega}}) = (1-e^{-j\hat{\omega}})^3 = e^{-j3\hat{\omega}/2}(-8j)\sin^3(\frac{\hat{\omega}}{2})$

$\mathcal{S}_5$: $H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$ $\Rightarrow$ $H(e^{j\hat{\omega}})$ has zeros at $\hat{\omega} = 0, \frac{\pi}{2}, -\frac{\pi}{2}$

$\mathcal{S}_6$: $y[n] = x[n] - x[n-3]$ $\Rightarrow$ $H(e^{j\hat{\omega}})$ would be zero at $\hat{\omega}=0, \frac{2\pi}{3}, -\frac{2\pi}{3}$

$\mathcal{S}_7$: $y[n] = \displaystyle\sum_{k=0}^{3} x[n-k]$ $\Rightarrow$ $H(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}/2}\dfrac{\sin 2\hat{\omega}}{\sin \hat{\omega}/2}$ $\Rightarrow$ zeros: $\hat{\omega} = \pi, \frac{\pi}{2}, -\frac{\pi}{2}$

$\mathcal{S}_8$: $y[n] = x[n] + 2x[n-1] + 3x[n-2] + 2x[n-3] + x[n-4]$