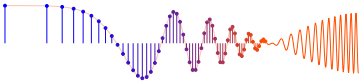


PROBLEM:

When a discrete-time signal is played out through a D/A converter and a speaker, we hear sounds over a well-defined frequency range. For this problem assume that the rate of the D/A converter is 8 kHz.

(a) A digital sinusoid is created using the formula: $y[n] = 44\cos(0.1\pi n + 0.3\pi)$ and then played out through the 8 kHz D/A. What analog frequency (in Hertz) is heard?

(b) A digital chirp signal is synthesized according to the following formula:

$$x[n] = \Re e\{e^{j\theta[n]}\} = \cos(0.0003\pi n^2) \qquad \text{for } n = 0, 1, 2, \ldots, 4000$$

Make a plot of the instantaneous *analog* frequency (in Hertz) versus time for the analog signal. Make sure that you determine the length of the signal in seconds. Does aliasing affect your answer?

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

When a discrete-time signal is played out through a D/A converter and a speaker, we hear sounds over a well-defined frequency range. For this problem assume that the rate of the D/A converter is 8 kHz.

(a) A digital sinusoid is created using the formula: $y[n] = 44\cos(0.1\pi n + 0.3\pi)$ and then played out through the 8 kHz D/A. What analog frequency (in Hertz) is heard?

$y[n] \rightarrow$ [D/A] $\rightarrow y(t)$ REPLACE n with 8000t

$y(t) = 44\cos(0.1\pi(8000)t + 0.3\pi)$.

$\therefore F_{OUT} = \frac{0.1\pi(8000)}{2\pi} = 0.05 \times 8000 = 400\,Hz$

(b) A digital chirp signal is synthesized according to the following formula:

$$x[n] = \Re e\{e^{j\theta[n]}\} = \cos(0.0003\pi n^2) \qquad \text{for } n = 0, 1, 2, \ldots, 4000$$

Make a plot of the instantaneous *analog* frequency (in Hertz) versus time for the analog signal. Make sure that you determine the length of the signal in seconds. Does aliasing affect your answer?

$x(t) = \cos(3\times10^{-4}\pi(8000t)^2)$

Let $\psi(t)$ = phase

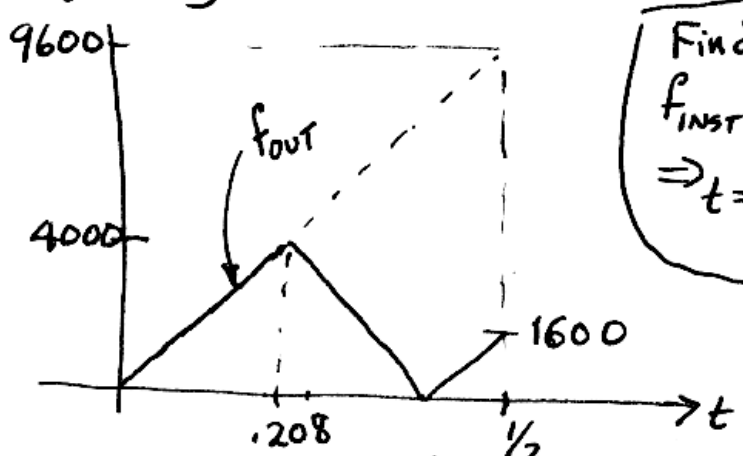
$\Rightarrow \psi(t) = 3\times10^{-4}\pi(64\times10^6)t^2 = 192\times10^2\pi t^2$

$f_{INST} = \frac{1}{2\pi}\frac{d}{dt}\psi(t) = \frac{1}{2\pi}(19200)2\pi t = 19200t$

when $n = 0, 1, \ldots 4000$, t goes from 0 to $t = \frac{1}{2}$

$f_{INST}(t) = 19200t\big|_{t=1/2} = 9600\,Hz$ which aliases.

Anything above 4000 Hz will alias.



Find t when $f_{INST}(t) = 4000$

$\Rightarrow t = \frac{4000}{19200} = .208$