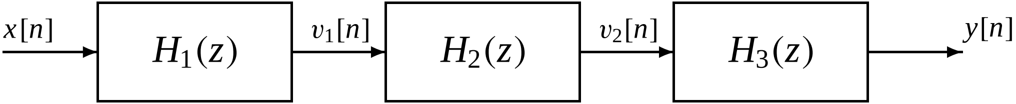
PROBLEM:

In the following cascade of systems, all systems are defined by their transfer functions.



$$H_1(z) = \frac{5}{1 - \frac{1}{3}z^{-1}} \qquad H_2(z) = \frac{2}{1 - \frac{1}{2}z^{-1}} \qquad H_3(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}$$

(a) Determine the unknown coefficients $\{b_k\}$ so that the output signal $y[n]$ will be the delayed impulse, $\delta[n-1]$, when the input signal $x[n]$ is an impulse, i.e., $x[n] = \delta[n]$.

(b) Using part (a), determine whether the following statement is true or false:

"For any input signal $x[n]$, the output is always $y[n] = x[n-1]$"

Give a solid reason to back up your choice of true or false.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

In the following cascade of systems, all systems are defined by their transfer functions.

$$x[n] \longrightarrow \boxed{H_1(z)} \xrightarrow{v_1[n]} \boxed{H_2(z)} \xrightarrow{v_2[n]} \boxed{H_3(z)} \longrightarrow y[n]$$

$$H_1(z) = \frac{5}{1 - \frac{1}{3}z^{-1}} \qquad H_2(z) = \frac{2}{1 - \frac{1}{2}z^{-1}} \qquad H_3(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}$$

(a) Determine the unknown coefficients $\{b_k\}$ so that the output signal $y[n]$ will be the delayed impulse, $\delta[n-1]$, when the input signal $x[n]$ is an impulse, i.e., $x[n] = \delta[n]$.

To get a delay by one, we need

$$H_1(z)\,H_2(z)\,H_3(z) = z^{-1}$$

$$\Rightarrow H_3(z) = z^{-1}\left(\frac{1}{H_1(z)}\right)\left(\frac{1}{H_2(z)}\right)$$

$$= z^{-1}\left(\frac{1 - \frac{1}{3}z^{-1}}{5}\right)\left(\frac{1 - \frac{1}{2}z^{-1}}{2}\right)$$

$$= \frac{z^{-1}}{10}\left(1 - \tfrac{5}{6}z^{-1} + \tfrac{1}{6}z^{-2}\right)$$

$$= \frac{1}{10}z^{-1} - \frac{1}{12}z^{-2} + \frac{1}{60}z^{-3}$$

$$\Rightarrow b_0 = 0$$
$$b_1 = 1/10$$
$$b_2 = -1/12$$
$$b_3 = 1/60$$

(b) Using part (a), determine whether the following statement is true or false:

"For any input signal $x[n]$, the output is always $y[n] = x[n-1]$"

Give a solid reason to back up your choice of true or false.

If the overall cascaded system is $H(z) = z^{-1}$

$$x[n] \longrightarrow \boxed{z^{-1}} \longrightarrow y[n]$$

then the difference equation is

$$y[n] = x[n-1]$$

$\boxed{\text{TRUE}}$