



## PROBLEM:

Evaluate the following and give the answer in both rectangular and polar form. In all cases assume that  $z_1 = -4 + j2$  and  $z_2 = -1 - j$ .

(a)  $z_1^*$

(d)  $jz_2$

(g)  $z_2/z_1$

(b)  $z_2^2$

(e)  $z_1^{-1} = 1/z_1$

(h)  $z_1 z_1^*$

(c)  $z_1 + z_2^*$

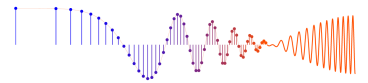
(f)  $z_1/z_2$

(i)  $z_1 z_2$

**NOTE:**  $z^*$  means the “conjugate” of  $z$ .

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.  
Prentice Hall, Upper Saddle River, NJ 07458. © 2003 Pearson Education, Inc.

**SOLUTION**



$$(a) z_1^* = (-4+j2)^* = -4-j2 = 4.472 e^{-j0.852\pi} = z_a$$

$$(b) z_2^2 = (-1-j)^2 = (-1)^2 + (-j)^2 + 2(-1)(-j) = 1-1+j2 = j2 = 2e^{j\pi/2}$$

$$(c) z_1 + z_2^* = (-4+j2) + (-1-j)^* = (-4-1) + j(2+1) \quad z_b$$

$$z_c = -5 + j3 = 5.831 e^{j0.828\pi} \approx 5.831 \angle 149^\circ$$

$$(d) jz_2 = z_d = j(-1-j) = 1-j = \sqrt{2} e^{-j\pi/4}$$

$$(e) z_e = 1/z_1 = \frac{1}{-4+j2} = \frac{z_1^*}{|z_1|^2} = \frac{-4-j2}{4^2+2^2} = \frac{-4}{20} - j\frac{2}{20}$$

$$= -0.2 - j0.1 = 0.224 e^{-j0.852\pi} \approx 0.224 \angle -153.4^\circ$$

$$(f) z_f = z_1/z_2 = \frac{z_1 z_2^*}{|z_2|^2} = \frac{(-4+j2)(-1+j)}{2} = \frac{(4-2) + j(-2-4)}{2}$$

$$= 1 - j3 = 3.162 e^{-j0.398\pi} \approx 3.162 \angle -71.6^\circ$$

$$(g) z_g \uparrow e^{z_2} = e^{-1-j} = e^{-1} e^{-j} = \frac{1}{e} (\cos(-1) + j\sin(-1))$$

$$= 0.1488 - j0.3096 = 0.3679 e^{-j0.318\pi} \quad \text{phase} = -1 \text{ radian}$$

$$(h) z_1 z_1^* = (-4+j2)(-4-j2) = 16 + 4 + j(-8+8) = 20$$

$$(i) z_1 z_2 = (-4+j2)(-1-j) = 4 + 2 + j(-2+4) = 6 + j2$$

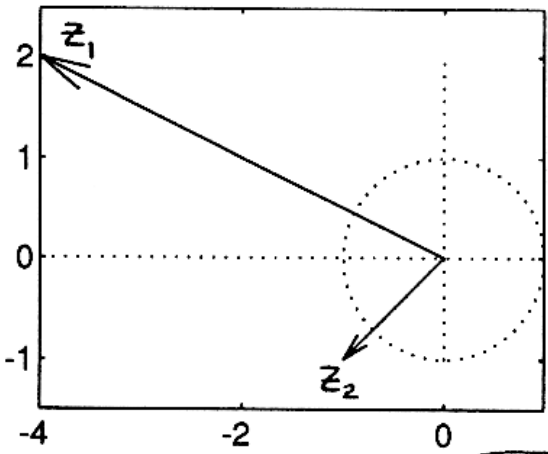
$$z_i = 6 + j2 = 6.325 e^{j0.102\pi} \approx 6.325 \angle 18.43^\circ$$

All these vectors are plotted using the labels  $z_a, z_b, z_c, \dots, z_i$  for the solution each part.

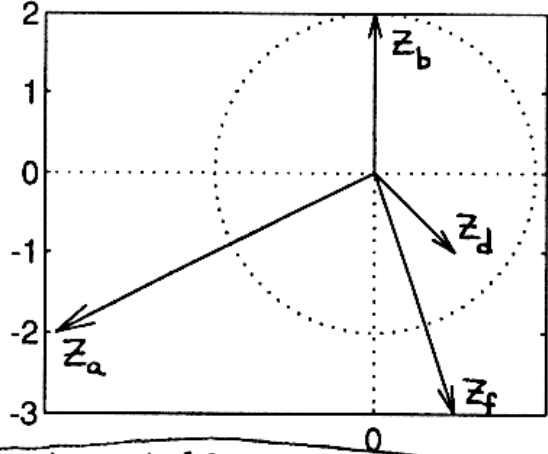


$Z =$	X	+ jY	Magnitude	Phase	Ph/pi	Ph(deg)
$z_1$	-4	2	4.472	2.678	0.852	153.43
$z_2$	-1	-1	1.414	-2.356	-0.750	-135.00
$z_a$	-4	-2	4.472	-2.678	-0.852	-153.43
$z_b$	0	2	2	1.571	0.500	90.00
$z_c$	-5	3	5.831	2.601	0.828	149.04
$z_d$	1	-1	1.414	-0.785	-0.250	-45.00
$z_e$	-0.2	-0.1	0.2236	-2.678	-0.852	-153.43
$z_f$	1	-3	3.162	-1.249	-0.398	-71.57
$z_y$	0.1988	-0.3096	0.3679	-1.000	-0.318	-57.30
$z_h$	20	0	20	0.000	0.000	0.00
$z_i$	6	2	6.325	0.322	0.102	18.43

PHASOR DIAGRAM

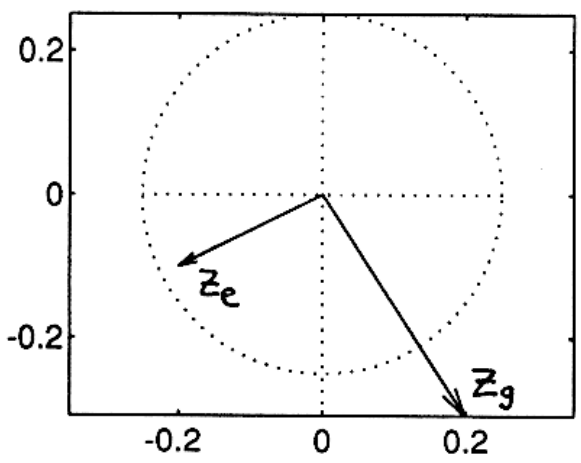


(a,b,d,f): PHASOR DIAGRAM



Note: each plot has different size

(e,g): PHASOR DIAGRAM



(c,h,i): PHASOR DIAGRAM

