PROBLEM:

Evaluate the following and give the answer in rectangular form.

(a) j^{37} (b) $e^{j(\pi/2+m\pi)}$ (m an integer)

(c) $j^{1/5}$ (find 5 answers) (d) $(1-j)^{1+j}$ (is the answer unique?)

McClellan, Schafer and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $j^{37} = \left(e^{j\pi/2}\right)^{37} = e^{j37\pi/2} = e^{j18.5\pi} = e^{j\pi/2} = j$

(b) $e^{j(\pi/2 + m\pi)} = e^{j\pi/2}\left(e^{j\pi}\right)^m = j(-1)^m = 0 + j(-1)^m$

(c) $j^{1/5} = \left(e^{j\pi/2}\right)^{1/5} = e^{j\pi/10}$

BUT WE CAN FIND MORE ANSWERS!

$e^{j\pi/2} = e^{j\pi/2}e^{j2\pi\ell} \qquad \ell = 0, \pm1, \pm2, \ldots$

$\therefore\ j^{1/5} = \left(e^{j(\pi/2 + 2\pi\ell)}\right)^{1/5} = e^{j(\pi/10 + 2\pi\ell/5)} \qquad \ell = 0,1,2,3,4$

ONLY FIVE ARE DIFFERENT:

$\{1\angle 18^\circ,\ 1\angle 90^\circ,\ 1\angle 162^\circ,\ 1\angle 234^\circ,\ 1\angle -54^\circ\}$

(d) $(1-j)^{1+j} = \left(\sqrt{2}\,e^{-j\pi/4}\right)^{1+j} = \left(\sqrt{2}\right)^{1+j} e^{-j\pi/4(1+j)}$

$= \sqrt{2}\left(\sqrt{2}\right)^{j} e^{-j\pi/4} e^{+\pi/4}$

THIS TERM PRESENTS A DIFFICULTY

Use "complex logarithm" $a^j = \left(e^{\ln a}\right)^j = e^{j\ln a}$

Since $\ln a = \ln|a| + j(\angle a + 2\pi\ell)$ BUT a is real, so $\angle a = 0$

$\therefore\ a^j = e^{j\ln|a|}e^{-2\pi\ell}$ for $\ell = 0, \pm1, \pm2, \ldots$

Take the case $\ell = 0$ to get one possible answer

$(1-j)^{1+j} = \sqrt{2}\,e^{j\ln\sqrt{2}}e^{-j\pi/4}e^{+\pi/4}$

$= 3.1018\,e^{j0.11\pi}e^{-j0.25\pi}$

$= 3.102\,e^{-j0.14\pi} = 2.808 - j1.318$

The answer is <u>NOT</u> UNIQUE