

PROBLEM:

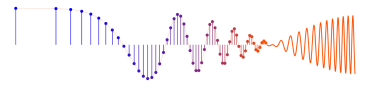
Evaluate the following and give the answer in rectangular form.

(a) j^{37}

(c) $j^{1/5}$ (find 5 answers)

(b) $e^{j(\pi/2+m\pi)}$ (m an integer)

(d) $(1-j)^{1+j}$ (is the answer unique?)



$$(a) \quad j^{37} = (e^{j\pi/2})^{37} = e^{j37\pi/2} = e^{j18.5\pi} = e^{j\pi/2} = j$$

$$(b) \quad e^{j(\pi/2 + m\pi)} = e^{j\pi/2} (e^{j\pi})^m = j(-1)^m = 0 + j(-1)^m$$

$$(c) \quad j^{1/5} = (e^{j\pi/2})^{1/5} = e^{j\pi/10}$$

BUT WE CAN FIND MORE ANSWERS!

$$e^{j\pi/2} = e^{j\pi/2} e^{j2\pi l} \quad l = 0, \pm 1, \pm 2, \dots$$

$$\therefore j^{1/5} = (e^{j(\pi/2 + 2\pi l)})^{1/5} = e^{j(\pi/10 + 2\pi l/5)} \quad l = 0, 1, 2, 3, 4$$

ONLY FIVE ARE DIFFERENT:

$$\{1 \angle 18^\circ, 1 \angle 90^\circ, 1 \angle 162^\circ, 1 \angle 234^\circ, 1 \angle -54^\circ\}$$

$$(d) \quad (1-j)^{1+j} = (\sqrt{2} e^{-j\pi/4})^{1+j} = (\sqrt{2})^{1+j} e^{-j\pi/4(1+j)} \\ = \sqrt{2} (\sqrt{2})^j e^{-j\pi/4} e^{+\pi/4}$$

THIS TERM PRESENTS A DIFFICULTY

Use "complex logarithm" $a^j = (e^{\ln a})^j = e^{j \ln a}$

Since $\ln a = \ln |a| + j(\angle a + 2\pi l)$ BUT a is real, so $\angle a = 0$

$$\therefore a^j = e^{j \ln |a|} e^{-2\pi l} \quad \text{for } l = 0, \pm 1, \pm 2, \dots$$

Take the case $l=0$ to get one possible answer

$$(1-j)^{1+j} = \sqrt{2} e^{j \ln \sqrt{2}} e^{-j\pi/4} e^{+\pi/4}$$

$$= 3.1018 e^{j0.11\pi} e^{-j0.25\pi}$$

$$= 3.102 e^{j0.14\pi} = 2.808 - j1.318$$

The answer is NOT UNIQUE